THE EFFECTS OF REPRESENTATIONAL SYSTEMS ON THE LEARNING OF STATISTICS BETWEEN GREEK PRIMARY SCHOOL STUDENTS AND IMMIGRANTS

Sofia Anastasiadou

University of Western Macedonia

The main objective of this study is to contribute to the understanding of the role of the four different types of representations and translations in statistical problem solving (SPS) in Greek primary school. Specifically, this study investigates the abilities of 3rd, 5th and 6th grade primary school indigenous students and immigrants in using representations of basic statistical concepts and in moving from one representation to another. The samples of the studies consisted of students of primary schools in Greece. The results of this study reveal that indigenous students have not acquired sufficient abilities for transformation from one representation system to another. Results reveal the differential effects of each form of representation on the performance of the two groups of students and the improvement of performance with age of indigenous students.

INTRODUCTION AND THEORETICAL FRAMEWORK

The notion of representations

In the field of statistics learning and instruction, representations play an important role as an aid for supporting reflection and as a means in communicating statistical ideas. Furthermore the NCTM's Principles and Standards for School Mathematics (2000) document include a new process standard that addresses representations and stress the importance of the use of multiple representations in statistical learning.

A representation is defined as any configuration of characters, images, concrete objects etc., that can symbolize or "represent" something else (Goldin, 1998). Representations have been classified into two interrelated classes: external and internal (Goldin, 1998). Internal representations refer to mental images corresponding to internal formulations that we construct of reality. External representations concern the external symbolic organizations representing externally a certain mathematical reality. In this study the term "representations" is interpreted as the "external" tools used for representing statistical ideas such as tables and graphs (Confrey & Smith, 1991). By a translation process, we mean the psychological processes involving the moving from one mode of representation to another (Janvier, 1987). Several researchers in the last two decades addressed the critical problem of translation between and within representations, and emphasized the importance of moving among multiple representations and connecting those (Gagatsis & Elia, 2004). Duval (2002) claimed that the conversion of a mathematical concept from one representation to another is a presupposition for successful problem solving. According to Elia and Gagatsis (2006) the role of representations in mathematical

J.F. Matos, P. Valero & K. Yasukawa (Eds.) (2008). *Proceedings of the Fifth International Mathematics Education and Society Conference*. Lisbon: Centro de Investigação em Educação, Universidade de Lisboa – Department of Education, Learning and Philosophy, Aalborg University.

understanding and learning is a central issue of the teaching of mathematics. The most important aspect of this issue refers to the diversity of representations for the same mathematical concept, the connection between them and the conversion from one mode of representation to others. Gagatsis and Shiakalli (2004) and Ainsworth (2006) suggest that different representations of the same concept complement each other and contribute to a more global and deeper understanding of it.

The understanding of a mathematical concept presupposes the ability to recognise the concept when it is presented with a series of qualitatively different representation systems, the ability to flexibly handle the concept in the specific representation systems and finally, the ability to translate the concept from one system to another (Lesh, Post & Behr, 1987). In statistical education, the interest focuses both on the various types of representation and on the translations between them.

This study intends to shed light on the role of different modes of representation on the understanding of some basic concepts in statistics. The study was designed to explore primary school students' performance in using multiple representations of statistical concepts with emphasis on the effects exerted on performance and on the relations among the various conversion abilities from one representation to another both by the age of the students (Anastasiadou, Elia & Gagatsis, 2007) and between indigenous students and immigrants.

The situation in Greece

At this point it is needed to give the situation in Greece in relation to immigrants' students because the cultural and ethnic elements are fundamentals to students' adjustment and progress in Education, Since the 1980s Greek society was one of the most homogeneous societies racially, culturally and linguistically. From that time until now there has been a continual but fluctuating influx of "foreigners" of contrasting characteristics: economic immigrants from ex-soviet union countries after the collapse of that state, new refugees (Greeks originating from the Pontos) from ex-soviet countries, economic immigrants and political refugees from mostly eastern islamic countries. This mainly economic immigration created a new reality of inequality of multilinguil, multiculturality in a country, or rather in a nation-state remarkably homogeinic linguistically and culturally (Kogidou, Tressou & Tsiakalos, 1997). In recent years Greek society has faced a particular challenge: *to create the right educational conditions for Greeks returning from abroad, foreign immigrants, romanies and muslims.* The de facto multiculturalism (Anastasiadou, 2007) which now describes the Greek society, as it does other countries, dictates the necessity to take on board these new approaches in education, society, in interstate relations and in cooperation since Greek society despite its multicultural character, continues to function with the logic of assimilation (Centre of Intercultural Education, 1998). In the field of education the adoption of the policy of assimilation means that it continues to have a monolingual and monocultural approach in order that every pupil is helped to acquire competence in the dominant language and the dominant culture. The attendance of children in Greek school with a different cultural or linguistic

expression is seen as a problem and must be discouraged. However, the problem is focused on the inability of those children to see the official language at school since it is thought that the learning of the official language is the basic ticket for their assimilation and academic achievement.

METHOD

Participants

The sample of the study involved 220 third grade indigenous students and 178 immigrants, 225 fifth grade indigenous students and 216 immigrants and 229 sixth grade indigenous students and 218 immigrants from primary schools in four regions of Western Macedonia. Below we briefly describe the content of teaching that students receive on statistics in the third, fifth and sixth grade of primary school according to the Greek curriculum, in order to give some information on students' prior knowledge.

The content of statistics in the third, fifth and sixth grade

According to the curriculum, third grade primary school students are taught to: record data, portray data through the relevant graphic and tabular representations, make assumptions and predictions regarding the results of the relevant actions, and reach the relevant conclusions based on the data. Additionally, according to the aims of the curriculum, third grade students must be able to: perceive the concepts of chance and probability, as well as the relationship between them, detect probable events, calculate the frequency of events and categorise the relevant statistical data and create the relevant tables.

Fifth grade students have often come across the terms "mean value" and "average" in math problems and can perhaps understand their meanings intuitively. According to the curriculum, fifth grade primary school students are taught: the meanings of the terms "average" and "mean value", how to read simple statistical tables and charts, how to use charts in order to present specific statistical data and how to empirically interpret the meaning of research. According to the aims of the curriculum, fifth grade students are expected to: understand the meaning and process of finding the average of the numbers provided; know the concepts of research, research population, research sample and research conclusions; know the basic steps that are required for conducting research, which include recording statistical data, sorting data, working out the absolute and relative frequency, graphic representations, calculating the average and formulating predictions and conclusions. Students themselves are required to gather and present statistical data that are drawn from their school and wider social environment.

In sixth grade students are taught to: record data, read simple statistical tables and charts, portray data through relevant graphic and tabular representations, read a table, extract information from it and convert it to a verbal or tabular representation, calculate the average and formulate predictions and conclusions. Additionally,

according to the aims of the curriculum, sixth grade students must be able to: work out the absolute and relative frequency, calculate the frequency of events and categorise the relevant statistical data, construct the relevant tables and bar or pie or histogram charts, understand the meaning and process of finding the average of the numbers provided.

Tasks and variables

A test was developed and administered to the students of the three grades. The test consisted of 6 tasks on frequency tables, bar charts and their application to solving everyday problems. These 6 tasks can be divided into three groups of two "equivalent" problems in difficulty from the mathematical point of view. In particular, the first task gives some information in verbal form and students are required to give the graphic form of this information (bar chart) (V1vg), while the second task gives information of the same kind in verbal form and requires its transformation into tabular form (V2vt). The second task is the following: "The values that follow represent the height of six children: Maria 100cm, Nicos 120cm, Kostas 132cm, John 140cm, Ann 114cm. Represent these data on a table." The third task involves reading a table (see Table 1) of the frequency of students' grades, extracting information from it and giving an interpretation in verbal form (V3tv).

Grade	6	7	8	9	10
Frequency	1	3	4	6	5

Table 1. The table included in the third task of the test

The fourth task involves reading a bar chart, extracting information from it and giving an interpretation in verbal form (V4gv). The fifth task involves reading a bar chart, extracting information from it and converting it to a tabular representation (V5gt). The sixth task involves reading a frequency and relative frequency table, extracting information from it and converting it to a bar chart (V6tg).

Right and wrong or no answers were scored as 1 and 0, respectively. Students' responses to the tasks comprise the variables of the study which were codified by an uppercase V (variable), followed by the number indicating the exercise number. Following is the letter that signifies the type of initial representation (e.g. r=representation, t=table, g=graphic, v=verbal) and, lastly, comes the letter that signifies the type of final representation.

Data analysis

For the analysis of the collected data the similarity statistical method (Lerman, 1981) was conducted using a computer software called C.H.I.C. (Classification Hiérarchique, Implicative et Cohésitive) (Bodin, Coutourier & Gras, 2000). This method of analysis determines the similarity connections of the variables. In particular, the similarity analysis is a classification method which aims to identify in a set V of variables, thicker and thicker partitions of V, established in an ascending

manner. These partitions, when fit together, are represented in a hierarchically constructed diagram (tree) using a similarity statistical criterion among the variables. The similarity is defined by the cross-comparison between a group V of the variables and a group E of the individuals (or objects). This kind of analysis allows for the researcher to study and interpret in terms of typology and decreasing similarity, clusters of variables which are established at particular levels of the diagram and can be opposed to others, in the same levels. It should be noted that statistical similarities do not necessarily imply logical or cognitive similarities. The red horizontal lines represent significant relations of similarity.

RESULTS

Descriptive results

Table 2 presents the success rates of third, fifth and sixth grade indigenous students and immigrants in all types of conversions. The results show that older indigenous students performed better than younger ones over all types of tasks but there was only a slight improvement in success rate between younger and oldest immigrants' students. Further more the success rates between the two groups of students are differentiating. The success rate of indigenous students are much higher that the immigrants ones in the all the three grades.

Students' success in each grade varies across the different conversion tasks. Considering the lowest and the highest percentage in each grade, this variation decreases with age: third grade indigenous students, 22-36% and immigrants, 10-18%; fifth grade indigenous students, 48-59% and immigrants, 12-18%; sixth grade indigenous students, 75-82% and immigrants, 15-22%. These findings indicate that maturation and instruction of statistics help students carry out conversions of statistical concepts more successfully and treat representations more flexibly.

Tasks	Type of translation	Third grade success rate of indigenous students (%)	Third grade success rate of immigrants (%)	Fifth grade success rate of indigenous students (%)	Fifth grade success rate of immigrants (%)	Sixth grade success rate of indigenous students (%)	Sixth grade success rate of immigrants (%)
V1vg	Verbal - Graphic	33.15%,	11.65%	54.67%	14.298%	82.3%	15.13%
V2vt	Verbal - Tabular	23.5%	10.54%	62.3%	15.73%	76.5%	15.86%
V3tv	Tabular - Verbal	22.2%	10.32%	60.4%	11.74%	75.8%	15.32%
V4gv	Graphic - Verbal	30.16%	11.87%	48.4%	12.09%	79.1%	14.7%
V5gt	Graphic- Tabular	24.4%	13.45%	53.7%	13.68%	74.6%	17.73
V6tg	Tabular- Graphic	35.56%	17.92%	59.2%	18.36%	79.4%	21.83%

Table 2. Success rates of indigenous students and immigrants in the tasks

In order to examine in a more comprehensive way the differences between 3rd, 5th and 6th grade **indigenous students and immigrants** with regard to their performance in the various tasks and the interrelation of their responses, a comparison was made between similarity diagrams 1, 2 and 3 concerning the 3rd, 5th and 6th grade respectively.

Similarity analysis results

The similarity diagrams in this study concern the data of each grade separately, and allow for the arrangement of students' responses (V1vg, V2vt, V3tv, V4gv, V5gt, V6tg) to the tasks into groups according to their homogeneity.

Two clusters (Cluster A and B) of variables are identified in the left similarity diagram of third grade indigenous students' responses as shown in Figure 1. The strongest similarity occurs between variables V1vg and V6tg in Cluster A. It is suggested that indigenous students employed similar processes to construct a graph based on information given verbally or in a table. The similarity connection of the variables V1vg and V6tg to the variable V4gv reveals students' consistency as regards their performance in constructing a graph and their performance in drawing information from the graph and interpreting it verbally. Cluster B consists of the variables V2vt, V5gt and V3tv. It is suggested that students dealt consistently with the tasks that required the construction of a table based on information given in verbal or in graphic form, as well as, with the task involving the verbal interpretation of its data.

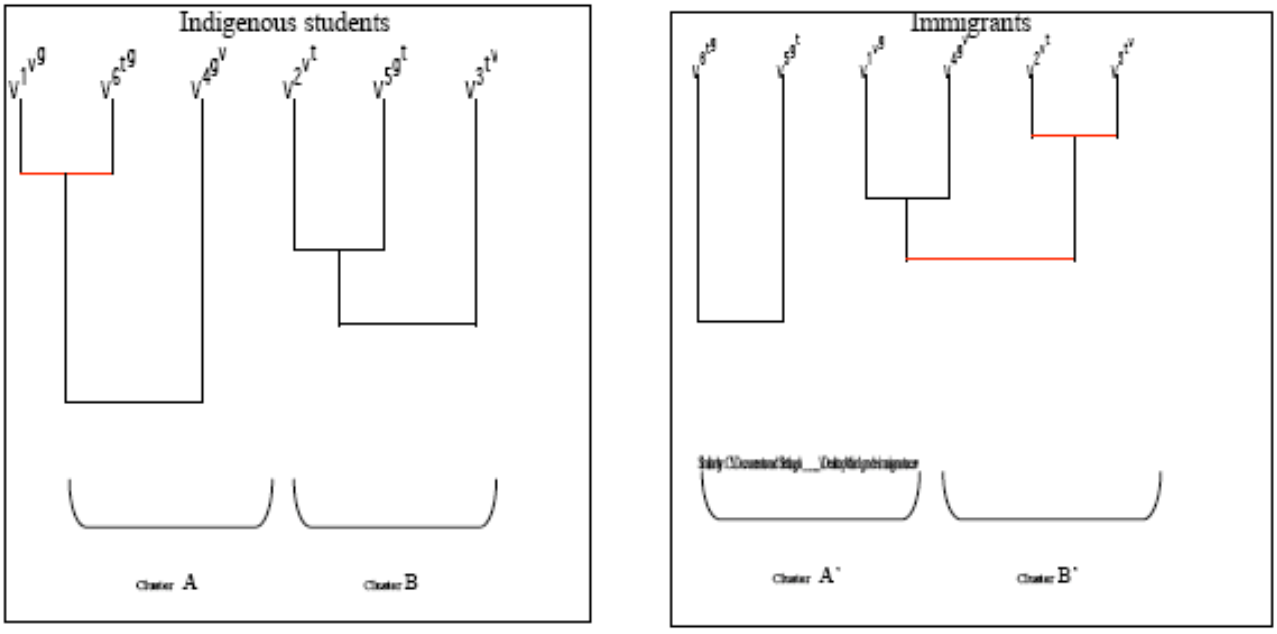

Figure 1. Similarity diagram of third grade indigenous students' and immigrants responses

The formation of the two distinct clusters indicates that indigenous students dealt differently with conversions requiring the construction of a graph or the verbal interpretation of a graph (V1vg, V6tg, V4gv), relatively to the conversions involving

the creation of a frequency table or a verbal description of the data given on a table (V2vt, V5gt, V3tv). This suggests that students in third grade treated the graphic and the tabular representations in isolation. Students' higher success rates at the tasks of the first cluster (V1vg: 33.15%, V6tg: 35.56%, V4gv: 30.16%) relatively to the tasks of the second cluster (V2vt: 23.5%, V5gt: 24.4%, V3tv: 22.2%) indicate their greater difficulty in tackling the second group of tasks and provide further support to the above assertions.

Two clusters (Cluster A' and B') of variables are identified in the right similarity diagram of third grade immigrants students' responses as shown in Figure 1. The strongest similarity occurs between variables V2tv and V3tv in Cluster B'. It is suggested that immigrants employed similar processes to construct a table based on information given verbally or to explain verbally the table elements. The similarity connection of the variables V2tv and V3tv to the variables V4gv and V1vg reveals students' consistency as regards their performance in constructing a graph and their performance in drawing information from the graph and interpreting it verbally and students' consistency as regards their performance in constructing a table and their performance in drawing information from the table and interpreting it verbally.

The formation of the two distinct clusters indicates that immigrants dealt differently with verbal conversions (V1vg, V4gv, V2vt, V3tv), relatively to the conversions involving the creation of a frequency table or a graph of the data given on a graph or a table (V6tg, V5gt). This suggests that immigrants in third grade treated the verbal representations in isolation. Students' higher success rates at the tasks of the second cluster (V6tg: 17.92%, V5gt: 13.45%) relatively to the tasks of the first cluster (V1vg: 11.65%, V4gv: 11.27%, V2vt: 10.54%, V3tv: 10.32%) indicate their greater difficulty in tackling the first group of tasks with verbal conversions and provide further support to the above assertions.

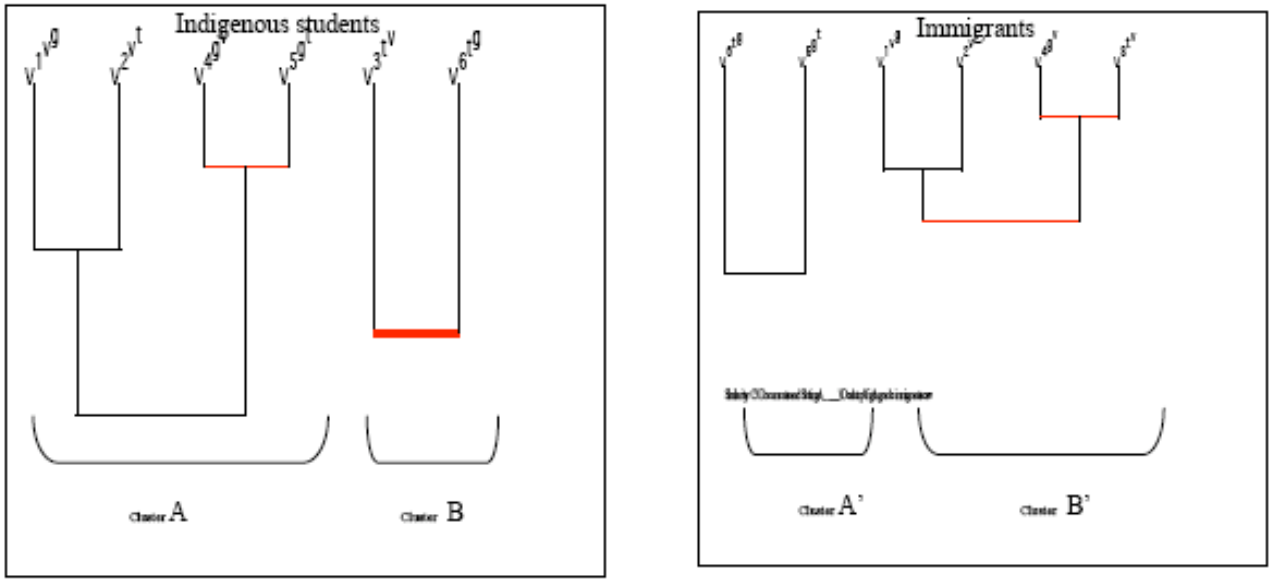

Figure 2. Similarity diagram of fifth grade indigenous students' and immigrants responses

The left similarity diagram of the fifth grade indigenous students' responses, illustrated in Figure 2, involves three pairs of variables (V1vg-V2vt, V4gv-V5gt, V3tv-V6tg). This grouping suggests that students dealt similarly with the conversions involving the same initial representation, which are verbal form, graph and table. Thus, the initial representation of the task had an effect on the conversion or interpretation processes employed by the fifth grade students. The similarity cluster (Cluster B) of the variables including the table as a starting representation (V3tv-V6tg) is disconnected from the other similarity pairs which form a joint cluster (Cluster A), indicating students' compartmentalized ways of handling frequency tables and the other forms of representation, i.e. graph and text.

The right similarity diagram of the fifth grade immigrants' responses, illustrated in Figure 2, involves three pairs of variables (V6tg-V5gt, V1vg-V2vt, V4gv-V3tv). The similarity cluster (Cluster A') (V6tg-V6gt) is disconnected from the other similarity pairs which form a joint cluster (Cluster B'), indicating immigrants' compartmentalized ways of handling verbal conversions and the other forms of conversions, i.e. graphical to tabular and tabular to graphical.

The strongest similarity in the similarity diagram of the sixth grade students' responses, illustrated in Figure 3, occurs between the variables V1vg and V6tg. This similarity reveals sixth grade students' consistency in their processes when constructing graphs on the basis of verbal or tabular representations. Students' responses to the other tasks are interwoven in the similarity diagram, indicating students' coherence in dealing with the corresponding conversions irrespectively of their initial or target representation. Students' high success rates at all of the tasks of the test ranging from 74.6% to 82.3% provide further evidence for this assertion.

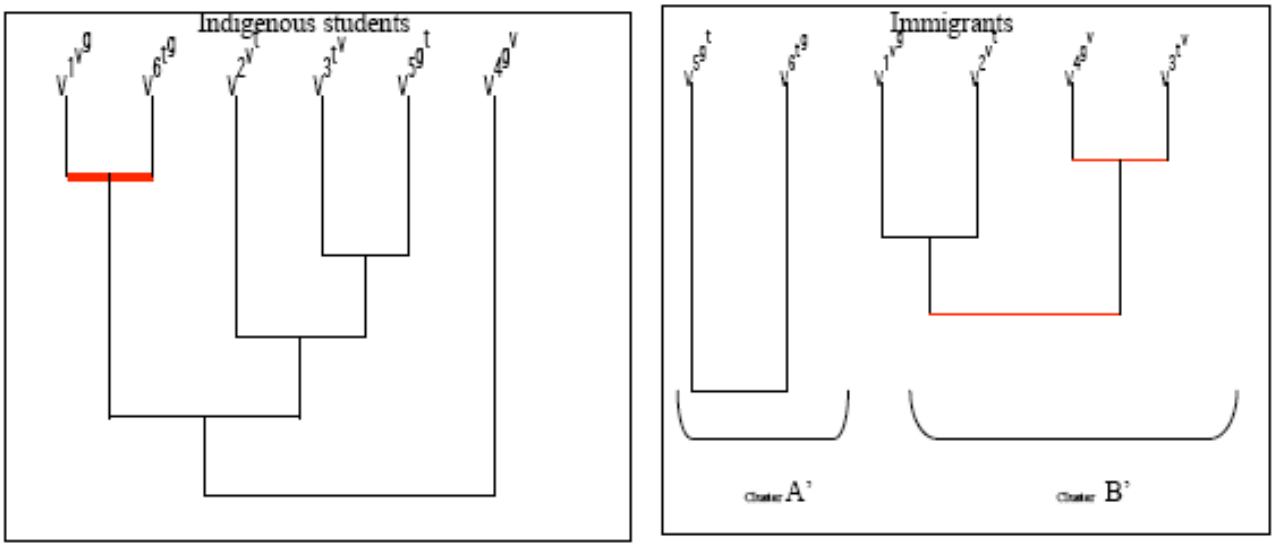

Figure 3. Similarity diagram of sixth grade students' responses

The right similarity diagram of the sixth grade immigrants' responses, illustrated in Figure 3, involves three pairs of variables (V5gt-V6tg, V1vg-V2vt, V4gv-V3tv). This grouping suggests that immigrants' responses are similar with the grouping in Figure

2 concerning immigrants' responses in fifth grade in conversions involving the same initial representation, which is verbal form, graph and table.

CONCLUSIONS

Representations are considered to be extremely important with respect to cognitive processes in developing statistical concepts. The main contribution of the present study is the identification of Greek indigenous students' and immigrants abilities to handle various representations, and to translate among representations related to the same statistical relationship across three age levels in primary education. Indigenous students' success was found to increase with age. Moreover, the three similarity diagrams clearly showed the different ways in which third, fifth and sixth grade students dealt with tasks involving different representations of statistical concepts.

These findings show that despite the improvement of students' performance from third to fifth grade, students in both grades encountered difficulties in the understanding of statistical concepts and more specifically in moving flexibly from one representation to another. Lack of connections among different modes of representations indicates the difficulty in handling two or more representations in mathematical tasks. This incompetence is the main feature of the phenomenon of compartmentalization in representations, which was detected in both third and fifth grade students (Duval, 2002). The phenomenon of compartmentalization in representations had also appeared with immigrants in the all three grades. The main difference of immigrants in relation to indigenous students is that there was no improvement of performance with age.

This phenomenon did not appear in the performance of sixth grade indigenous students. Their success was found to be independent of the initial or the target representation of the tasks. The basic problem with immigrants is the understanding of the verbal representations as an initial and final mode and this due to the general difficulties that immigrants face with the Greek language.

REFERENCES

Anastasiadou, S., Elia I., Gagatsis A. 2007. *Apprentissages des statistiques te représentations chez L'élevé de l' Ecole Primaire de Grèce,* 4th International Meeting on Statistical Implicative Analysis, pp. 135-144.

Ainsworth, S.: 2006. 'DeFT: A conceptual framework for considering learning with multiple representations', *Learning and Instruction* 16, 183-198.

Bodin, A., Coutourier, R. and Gras, R.: 2000. '*CHIC : Classification Hierarchique Implicative et Cohésitive-Version sous Windows – CHIC 1.2'.* Association pour le Recherche en Didactique des Mathematiques, Rennes.

Christou C, Gagatsis, A. and Zachariades T.: 2001. 'The difficulty level of representations in mathematical relationships', *Scientia Paedagogica Experimentalis* XXXVIII(1), 25–38.

Confrey, J. and Smith, E.: 1991. 'A framework for functions: Prototypes, multiple representations and transformations', in R. G. Underhill (ed.), *Proceedings of the thirteenth annual meeting of the North American Chapter of the International Group for the Psychology of Mathematics Education,* Virginia Polytechnic Institute and State University, Blacksburg, pp. 57-63.

Duval, R.: 2002. 'The cognitive analysis of problems of comprehension in the learning of mathematics', *Mediterranean Journal for Research in Mathematics Education* 1(2), 1-16.

Elia, I. and Gagatsis, A.: 2006. 'The effects of different modes of representation on problem solving: Two experimental programs', in J. Novotná, H. Moraová, M. Krátká and N. Stehlíková (eds)., *Proceedings of the 30th Conference of the International Group for the Psychology of Mathematics Education,* Vol. 3, Prague, Czech Republic, pp. 3-25.

Gagatsis, A. and Elia, I.: 2004. 'The effects of different modes of representations on mathematical problem solving', in M. Johnsen Hoines and A. Berit Fuglestad (eds.), *Proceedings of the 28th Conference of the International Group for the Psychology of Mathematics Education,* Vol. 2, Bergen University College, Bergen, Norway, pp. 447-454.

Gagatsis, A. and Shiakalli, M.: 2004. 'Ability to translate from one representation of the concept of function to another and mathematical problem solving', *Educational Psychology* 24(5), 645-657.

Goldin, G.A.: 1998. 'Representational systems, learning, and problem solving in mathematics', *Journal of Mathematical Behavior* 17(2), 137-165.

Janvier, C.: 1987. 'Representation and understanding: The notion of functions as an example', in C. Janvier (ed.), Problems of representation in the teaching and learning of mathematics, Lawrence Erlbaum Associates, Hillsdale, N.J., pp. 67-72.

Kogidou D., Tressou E., Tsiakalos G. "Social exclusion and education: the case of the languages of the minorities in west Thessaloniki". Skourtou E. (eds.). Themes in Diglossa and education: Athens: Nisos. 1997.

Lerman, I. C.: 1981. 'Classification et analyse ordinale des données', Dunod, Paris.

Lesh, R., Post, T. and Behr, M.: 1987. 'Representations and translation among reprensentations in mathematics learning and problem solving', in C. Janvier (ed.), *Problems of Representation in the Teaching and Learning of Mathematics,* Lawrence Erlbaum, Hillsdale, NJ, pp. 33-40.

NCTM.: 2000. *'Principles and standards for school mathematics'*, NCTM, Reston, Va.