

CONTRADICTIONS IN MATHEMATICAL MODELLING ACTIVITIES FROM A CRITICAL MATHEMATICS EDUCATION PERSPECTIVE [1]

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In this article, I present an experience that took place during a Mathematics I course proffered to undergraduate geography students. The context is the presentation of a mathematical modelling project developed by a group of students enrolled in the course. The objective of the article is to apply an analysis, using Activity Theory, of apparent contradictions between students' views regarding the use of mathematics in geography, the guidance they received regarding the development of the mathematical modelling project, and the words of the group members during the final presentation of their project. The comments made by students during these different instances, which may appear contradictory, can be re-interpreted when analyzed in a broader activity, from the perspective of Activity Theory.

INTRODUCTION

The experience I describe here took place in the first semester of 2006 in a Mathematics I course offered to undergraduate students of the geography program at the Federal University of Minas Gerais (UFMG). Specifically, it refers to a presentation of a mathematical modelling project developed by one group of students enrolled in the course.

The objective of the article is to apply an analysis, using Activity Theory, of apparent contradictions with respect to three aspects (or at three instances): 1) students' views regarding the use of mathematics in the field of geography (to create the science of geography, to describe and discuss geographical phenomena, and as a required course within the undergraduate geography program); 2) the guidance they received regarding the development of the mathematical modelling project; and 3) the statements made by the group members during the final presentation of their project.

With the aim of presenting students' views regarding the use of mathematics in the field of geography, I begin by describing some characteristics of the Mathematics I course as well as of the students enrolled in it. In this way, I present the space in which the mathematical modelling project was developed. In the third section, I lay the theoretical groundwork for the perspective of mathematical modelling adopted and describe how it was carried out, emphasizing the guidance the students received regarding the development of the project. The project developed by one group of students is presented in the fourth section.

Some of the comments made by students during these different instances which appear, in principle, to be contradictory, can be re-interpreted when analyzed in a

broader activity, from the perspective of Activity Theory. In the fifth section, I attempt to apply such an analysis, suggesting possibilities for this re-interpretation.

THE COURSE AND ITS STUDENTS

Until March, 2006, Mathematics I was among the required courses for the undergraduate program in Geography at the UFMG. Based on the need for new pedagogical projects for the program to meet new curriculum guidelines for elementary teacher education (BRASIL, 2002), the course became optional. I was assigned by the Mathematics Department to teach the course in the first semester of 2006.

The mathematics contents planned for the course included functions, derivatives, and notions of integral. However, in the interest of developing the classes in accordance with the pace of the students, not all of this content was covered in 2006. With few exceptions, geography students generally have a history of poor relationships with mathematics, complaining of learning difficulties, traumatic past experiences, and disappointment regarding the requirement to take the course, having believed they were finally free from mathematics.

When I inquired about the role of Mathematics I within the Geography program, the students reacted with indignation. In addition to their reports of difficulties with mathematics in previous academic experiences, they argued that geography pertains to the Human Sciences, and as such, cannot be constructed in terms of mathematical arguments. At times they confused the use of mathematics in geography with a positivist approach to the latter, and armed with critiques of Positivism, they questioned the validity of its use. Students required to repeat the course stated that they failed to see the applicability of Mathematics I in their program. Thus, in the students' opinion, Mathematics I should not be included in the curriculum of the Geography program. According to the coordinator of the program at that time,

students always complained that they were unable to see mathematics in a practical way in their academic future (connection with other courses) and their professional futures. Remaining distant from the reality of geography, mathematics loses its meaning, in their opinions (MAGALHÃES JR., 2006).

And, in fact, the Mathematics I course was removed from the required curriculum of the Geography program.

Regarding the activities developed in the first semester of 2006, due to a study that was in the phase of data collection at that time (ARAÚJO & PINTO, 2004) [2], I proposed the development of *milieus of learning* with computers within which the students were invited into a *landscape of investigation* (ALRØ & SKOVSMOSE, 2002). But since the course was being offered for the last time, and based on a positive previous experience (ARAÚJO, 2004), I could not miss the opportunity to propose the development of mathematical modelling projects, described in greater detail in the following section.

THE DEVELOPMENT OF MATHEMATICAL MODELLING PROJECTS

Mathematical modelling has stood out among current perspectives in mathematics education. In general terms, it can be understood as the utilization of mathematics to resolve real problems. When applied in the classroom, this approach takes on special forms, depending on the educational context, the professionals involved, and the profile of the students, among other factors.

Bassanezi (2002), for example, understands mathematical modelling – whether as a scientific method or a teaching and learning strategy – as the “art of transforming problems from reality into mathematical problems and resolving them through interpretation of their solutions in the language of the real world” (p. 16). For Barbosa (2001), “modelling is a milieu of learning in which students are invited to question and/or investigate, by means of mathematics, situations with reference in reality” (p. 31).

In the Mathematics I course, I sought to put into practice an understanding of mathematical modelling as

an approach, by means of mathematics, to a non-mathematical problem based in reality, or to a non-mathematical situation based in reality, chosen by groups of students in such a way that questions of Critical Mathematics Education form the basis for the development of the work (ARAÚJO, 2002, p. 39).

Within this perspective, there are some explicit characteristics of the milieu of learning that I seek to put into effect when I propose the development of mathematical modelling projects, including working in groups, and basing the work on Critical Mathematics Education.

According to Skovsmose (1994), the main concern of Critical Mathematics Education is the development of *mathemacy*, which is an extension to mathematics of the problematizing and liberating conception of education proposed by Freire (1970). A similar concept – *matheracy* – has also been discussed by D’Ambrosio (1999). In *mathemacy*, the objective is not to merely develop the ability to carry out mathematical calculations, but also to promote the critical participation of students/citizens in society, discussing political, economic, and environmental issues in which mathematics serves as a technological support. In this case, critique is directed at mathematics itself, as well its use in society, the concern thus extending beyond the teaching and learning of mathematics.

The development of the modelling project in the Mathematics I course began with the discussion of a text (ARAÚJO, 2006). In this text, I present my understanding of mathematical modelling and suggestions for topics that should be considered in the “research proposal” to be written by the groups. At the same time, students were asked to think about themes for their projects and about the formation of groups to develop them.

In the following class, themes and groups were defined through a long process of negotiation. In the first semester of 2006, each group ended up with approximately seven members, and the themes chosen were the following: the transposition of the São Francisco River (two groups formed, one to address physical aspects and the other social aspects); physical impacts of the implantation of hydroelectric dams; socio-cultural aspects of the Linha Verde (Green Line) freeway construction project in Belo Horizonte; Campus 2000: consequences for transportation in the UFMG; climate myths; solar energy.

Once the themes had been defined, each group elaborated a work plan which I evaluated and returned to the group. In this evaluation, I encouraged them to describe in detail all the steps to be followed during the development of the project, as well as the definition of the focus of the research. I also sought to raise questions regarding how mathematics would be used in the project.

After the projects had been approved, the groups began to carry them out, holding meetings during and outside of class. They presented partial reports on their progress each month, and based on these reports, each group received guidance and suggestions - my own as well as from the entire class - regarding how to proceed. During each of these advisory sessions, I sought to take into account the concerns of Critical Mathematics Education.

At the end of the semester, all the groups made an oral presentation of their project to the class (which were videotaped), and handed in a written version of the project. One project, in particular, attracted my attention because of the group's careful treatment of the mathematical information. This project is considered in greater detail in the section that follows.

FINAL PRESENTATION OF THE PROJECT “TRANSPOSITION OF THE SÃO FRANCISCO RIVER: PHYSICAL ASPECTS”

The theme of the group's project was “physical aspects of the transposition of the São Francisco River”. The group's choice of theme portrays, at the same time, the relation with their field of interest, geography, and their interest in a controversial subject, the transposition of the São Francisco River [3]. The objective of the project was to analyze whether or not the rainfall in a given region along the course of the river would be sufficient to compensate for the amount of water that would be diverted as a result of the transposition.

This small report demonstrates the possibility for using mathematics (**quantity** of rainfall and diverted water) to discuss a problem from geography (**quantity of rainfall**) in a critical manner (questioning the environmental consequences). Thus, a mathematical modelling project was proposed that could be approached from a Critical Mathematics Education perspective.

In the written report [4], the group reported that, after agreeing on the objective of the research (analyze the quantity of rainfall . . .), they began to consider what mathematical model to use, and decided, without much justification, to adopt a periodic function. This choice may have resulted from the students' knowledge regarding the behaviour of rainfall, but it may also have been influenced by the subject discussed in class, which would exemplify what Araújo and Barbosa (2005) call the inverse strategy in the modelling process. However, data that the group had gathered, relating to the rainfall in a given region along the course of the river, seemed not to fulfil a periodic function. Then, the solution that the group found was to re-group data in such a way that they fit a mathematical model represented by a periodic function.

In my point of view, this is an example of what Skovsmose (1994) calls the *formatting power of mathematics*. The author defends the thesis that mathematics is used to format reality. According to this thesis, part of our reality is projected by means of mathematical models. One example of this is the Human Development Index (HDI): based on mathematical models, a number from zero to one is associated with every city or locale. Based on this index, the government, for example, decides how to distribute funds to achieve a given objective. A city with an HDI near 1, because of their relatively high rating, might not be selected to receive funds that could resolve some of their problems. Thus, mathematical models are used to create a "real situation" that did not exist before. Critical Mathematics Education questions this power with which mathematics is imbued.

In the case of the group of geography students, it appears to me that the data relating to the precipitation in the region they chose were formatted by a periodic model.

Moreover, the group appeared to construct certainties regarding the mathematical discussion they developed. In the written work, they state that the development of the modelling project was important for the group to agree "that the science of geography needs mathematical analysis **to prove** environmental or social impacts" (my emphasis).

Such statements reinforce what Borba and Skovsmose (1997) call *the ideology of certainty of mathematics*. According to these authors, the ideology of certainty sustains the character of neutrality of this science, imbuing it with the power of the holder of the definitive argument in various debates in society. Thus, mathematics is considered in the presentation of political decisions, for example, suggesting that the decision taken represents the best path to follow, without leaving room for counter-arguments, thus characterizing its use as a *language of power*. Combating the ideology of certainty is one of the objectives of Critical Mathematics Education.

On the other hand, as mentioned in section "The Course and Its Students", these same students are uncompromising critics of the use of mathematics to discuss social issues. They believe the exactness of mathematics to be insufficient to account for the complexity involved in subjects from the Human Sciences. In addition, the

development of the modelling projects was guided in such a way that the ideas of Critical Mathematics Education were considered, as described in section “The Development of Mathematical Modelling Projects”. In other words, I, the professor, expected the students to question the use of mathematics to generate certainties regarding the transposition of the São Francisco River, and to use mathematics as **one** way (and not **the** way) to understand the situation, and not to format the information they gathered. What was happening with the group? Could we say that the group was contradictory in the development of this mathematical modelling project?

APPARENT CONTRADICTIONS: APPLYING AN ANALYSIS

To say that there were contradictions in the group’s work during the three instances discussed here could be understood colloquially. In this case, we would say that there are incoherencies or conflicts in their statements. Understood in this manner, our evaluation of the group could have a negative connotation: the students were not very sure about what they were doing.

On the other hand, contradiction is a key concept in Activity Theory, as according to Engeström (1987), internal contradictions are “the source of dynamics and development in human activity.” The word “activity”, despite being part of our everyday vocabulary, is also the central concept to Activity Theory, which has its origins in the historical-cultural school of Soviet psychology, whose principle representative is Vygotsky.

Activity Theory considers activity as the basic unit of human development. According to Leont’ev (1978), it is born of the process of reciprocal transformations between subject and object. In his own words,

Activity is a molar, not an additive unit of the life of the physical, material subject. In a narrower sense, that is, at the psychological level, it is a unit of life, mediated by psychic reflection, the real function of which is that it orients the subject in the objective world. In other words, activity is not a reaction and not a totality of reactions but a system that has structure, its own internal transitions and transformations, its own development. (p. 50).

According to this reference, the contradictions emerge from the duality of human activity, as a production of society, in general, and as a specific production within an activity. This duality is the result of the relation between the individual and society (ROTH, 2004).

How, then, can we re-interpret the procedures of the group of geography students at the three instances presented, according to this perspective?

One possibility is to understand the development of the mathematical modelling project as an activity. If that were the case, according to Leont’ev (1978), it should have a motive, a need that drives it. Interpreting it in this way, the object of the activity of the group of geography students (the subjects of the activity) was the

transposition of the São Francisco River; and what drove them, their motive, was their questioning regarding the real need and conditions of the river for this procedure. Thus, mathematics could have been used, in a critical manner, as part of the analysis developed by the group.

On the other hand, the mathematical modelling project was one of the tasks to be used to evaluate students for the Mathematics I course; and if we interpret the course as an activity in which the students were subjects (together with the professor), perhaps their motive was to pass the course. Understood in this way, a whole set of values traditionally associated with school mathematics can come into play and influence the students' procedures during the development of the modelling project. For example, the students, who had a history of problems with mathematics in school, may want to show the professor that they now have command of this powerful tool and no longer view it with disregard.

It thus appears that isolated assignments that take into account questions raised by Critical Mathematics Education may have little influence on students who live in a social world that, in general, values the power of mathematics and rarely questions the reality it constructs.

We can, however, re-interpret the procedures of the group of geography students in a direction opposite that of the preceding analysis. Understood in this way, what was in principle an action within a broader activity aimed at passing the Mathematics I course became a new activity that took on its own life, acquired its own motive, and in this way, more closely approximated my initial intention of discussing problems or situations from geography, by means of mathematics, from a Critical Mathematics Education perspective.

I believe that it is important to develop work from a Critical Mathematics Education perspective in specific situations (in a classroom, for example), but aiming to extend this discussion to society. According to Activity Theory, the individual, as a social being, is influenced by the values, conceptions, traditions, etc., that are part of society, but at the same time, he/she has the power to change these values, acting (critically) in this same society.

NOTES

1. The author's attendance at MES5 was partially funded by Fundação de Amparo à Pesquisa do Estado de Minas Gerais (FAPEMIG). Although they are not responsible for the ideas presented in this paper, I would like to thank Alex Jordane, Caroline Passos and Diva Silva, postgraduate students from UFMG, for their comments on the original draft.
2. Research project developed with the support of the Conselho Nacional de Desenvolvimento Científico e Tecnológico (CNPq) and FAPEMIG, both Brazilian governmental agencies that support scientific research.
3. The São Francisco River is the most important river in the Brazilian Northeast, the driest region in the country. The fertile areas along the river contrast with the rest of the region, dominated by caatinga. For years, there has been talk in Brazil of diverting the waters of the river to other areas of the northeast. However, the river has been suffering from

pollution and silting, and it is not known whether it would withstand such a procedure. It is also known that there are political interests involved. In summary, it is a very controversial topic.

4. The complete reference of the group's written work is not presented to preserve their identity.

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