ARE MY STUDENTS ENGAGED WITH CRITICAL MATHEMATICS EDUCATION?

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Students enrolled in a pre-service Primary-Middle Years teacher education program at the University of South Australia undertake two integrated mathematics and science education courses. Many of the learning topics within the courses introduce students to a critical perspective of mathematics and science education. The author is a member of the team that constructs and teaches the courses and has recently commenced a PhD program with a critical mathematics education focus. Reading in the field of critical mathematics education has raised many questions about this focus in each course. This paper draws from Paul Ernest, Marilyn Frankenstein, Betty Johnston, Gelsa Knijnik and Ole Skovsmose to explore one of these questions, are students undertaking these courses really engaged in critical mathematics education?

Ernest (2001), Frankenstein (1998, 2000), Johnston (1994), Knijnik (1998, 2002), Skovsmose (1994, 2005) and Vithal (2003) have all written in the field of critical mathematics education. Each presents a particular construct for critical mathematics education. This paper comes from grappling with the concept of critical mathematics education, elements that constitute each of its various constructs and reflecting upon how these elements are part of learning topics within the integrated courses. The paper has four parts; a discussion about the role of critical mathematics education and four elements drawn from its various constructs, a description of the student and course context, a discussion about how one of the learning topics engages students with critical mathematics education and a conclusion.

CRITICAL MATHEMATICS EDUCATION

Critical mathematics education comes from the need to foreground the role of mathematics education in educating for citizenship and empowerment. Skovsmose (2005, p. 3) puts the view, "To acknowledge the critical nature of mathematics education, including all the uncertainties related to this subject, is a characteristic of *critical mathematics education*." Learning mathematics may result in empowerment, citizenship and democratic participation or it may result in disempowerment, marginalisation and exclusion. Critical mathematics education and ethnomathematics are two developments where some versions aim to implement a more equitable mathematics education and mathematics education for democracy and citizenship.

Critical mathematics education challenges the nature of mathematical knowledge, the purpose of mathematics education and, in turn, challenges mathematics curricula and classroom practices. Ernest (2001), Frankenstein (1998, 2000), Knijnik (1998, 2002), Skovsmose (1994, 2005) and Vithal (2003) amongst others argue that education policies, curricula and teaching practices present mathematics as an absolute, neutral,

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unquestioned body of knowledge that has little to do with the socio-political environments we live in and does little to develop students as participating, thinking citizens.

Ernest (2001, p. 278) argues that critical mathematics education is about applying a critical attitude to mathematics and its teaching. As such, a key aim for critical mathematics education is for "students to think mathematically, use it in their lives to empower themselves both personally and as citizens, and appreciate its role in history, culture and in the contemporary world" (Ernest, 2001, p. 285). Skovsmose (1994, p. 16) states to be critical "means to draw attention to a critical situation or a progression in a crisis, to identify it, to try to grasp it, to understand it and to react to it." In his philosophy for critical mathematics education, Skovsmose (1994) focuses on the role and formatting power of mathematics in highly technological societies and how these are often hidden and left unquestioned. Mathematics has made positive contributions to society but has created what Skovsmose (1994) and Skovsmose and Nielson (1996) describe as crises with respect to inequitable distribution and access to resources, democratic participation and acting ethically. Skovsmose (2005, pp. 4-5) further develops the notion of problematic outcomes from mathematics education with his discussion about the concerns of mathematics education, particularly the concern of globalisation and ghettoising and the emergence of a Fourth World.

The ideals and values that underpin critical mathematics education as discussed by Skovsmose and others support the principles of the Primary-Middle Years Program.[1] The ideals and values also support the teaching team in two ways. Firstly, through mathematics and science, engage students with issues from the perspectives of social justice, equity, sustainability and ethical action. Secondly, through interactive, collaborative learning topics, model practices that can implemented in classrooms.

The literature around critical mathematics education presents a complex, evolving concept with a range of views about what constitutes critical mathematics education. Johnson (1994) introduced the idea of critical numeracy as a way to make meaning with mathematics that extends beyond the functional or utilitarian application of mathematics to include socio-historical perspectives and the potential to explore the power relations when applying mathematics. Frankenstein (1998, 2000) developed the idea of criticalmathematics literacy based on interrogating the use of number and statistics in particular to interpret and challenge inequalities in society. Her criticalmathematics literacy curriculum framework is built around the four goals about understanding mathematics, the mathematics of political knowledge, politics of mathematical knowledge and the politics of knowledge (Frankenstein 1998, p. 1).

Mathemacy was first introduced by Skovsmose (1994) as a key competence within critical mathematics education that mirrors Freire's competence of literacy for democracy. Mathemacy is a competence with mathematics to interpret social life, to act in a world structured by mathematics (Skovsmose 1998, p. 200). It draws together three ways of knowing; mathematical, technical and reflective knowing with reflective knowing being the potential catalyst for critical awareness. Ernest (2001, pp. 285–286) describes what a critical mathematics education should encourage through five points of awareness or understanding that in many ways reflect the goals of politics of mathematical knowledge and mathematics of political knowledge (Frankenstein 1998) and reflect the ideas of mathemacy and mathematics in action with respect to science and technology (Skovsmose 1994, 1998, 2005).

Through her work with the Movimento dos Sem-Terra, Knijnik (1998, 2002) developed an ethnomathematics approach to empower students and community members. The approach identifies and recovers popular mathematical knowledge drawn from personal and community activities. The popular mathematics is shared and decoded. Knowledge is also acquired from academic mathematics and comparisons established between the popular and academic knowledge (Knijnik 1998, p. 189). A criticism of ethnomathematics is that while community practices are acknowledged and valued, the popular mathematics is glorified and exacerbates cultural relativism (Vithal & Skovsmose, 1997). Knijnik's ethnomathematics approach addresses this criticism as the power relations produced by the confrontation between popular and academic mathematics become the centre of comparison and analysis and as such opens up a larger world view.

Grappling with each of the above constructs for critical mathematics education has highlighted elements common to most constructs and others particular to a few constructs. Given the length of this paper, I have chosen four elements as the basis for answering my question. My intent is not to fragment elements in ways that loses the relationships between them nor do I claim to be exhaustive with my identification. The elements chosen are: authentic, interdisciplinary learning experiences; landscapes of investigations; the role of mathematics in socio-political contexts; and reflection, critique and action.

Authentic, interdisciplinary learning experiences

Engaging learners with authentic, interdisciplinary experiences is a common theme across the constructs of critical mathematics education. Examples of practice from each author engage learners with socio-political contexts that provide opportunities to problematise the application of mathematics and maximise drawing upon other disciplines. Frankenstein (1998) argues that mathematics is made more accessible through real-life contexts through an interdisciplinary mathematics and social studies curriculum. The ethnomathematics approach developed by Knijnik (1998) identifies, decodes and shares people's popular mathematics practices and continually engages them with the socio-political contexts from which they are drawn. Ernest (2001, pp. 289-291) offers examples drawn from social, political and environmental aspects of everyday life that have a local or a more global perspective as rich contexts to engage learners with critical mathematics education. This is also a characteristic of the project work described by Skovsmose (1994) and Arlø & Skovsmose (2002).

Landscapes of investigations

Closely connected to authentic, interdisciplinary learning experiences are landscapes of investigation. Landscapes of investigation engage students with questions that challenge and questions that lead to explanations, situations that offer more than one answer, support student directed investigation, enable students to develop their mathematics as part of their investigation, and encourage teachers to work in a risk zone where questions, directions and outcomes may change (Skovsmose, 2001a). The idea of landscapes of investigation is reflected in Frankenstein's criticalmathematics literacy framework and Knijnik's ethnomathematics approach. Frankenstein (1998) promotes engaging students with deep and complicated questions, open-ended investigations where students pose their own questions and create their own problems. Knijnik (1998) supports students to share, decode and compare their mathematical practices and explore thinking behind these practices.

The role of mathematics in socio-political contexts

A key tenet of critical mathematics education is that mathematics is applied in contexts of particular social, economic and political values, ideologies and vested interests. Johnston (1994, p. 35) argues that unless one asks, 'in whose interest is mathematics used?' and acts in response to this, one cannot be critically numerate. Frankenstein (1998) argues that students need to develop an understanding of the mathematics of political knowledge and the politics of mathematical knowledge. The former refers to how mathematics can be used to understand institutional structures in society while the latter argues that applying mathematics is not neutral and is underpinned with political positions or perspectives. Knijnik (1998) makes the power relations between popular and legitimised mathematics the centrepiece of her ethnomathematics approach to uncover the vested interests and impact of mathematics on societies. Skovsmose (1994, 2001b) puts the view that mathematical models are at the centre of the formatting power of mathematics in highly technological societies. Mathematical models are developed in the contexts of social, economic and political interests or as a result of further technological development. The interests behind and the impact of applying mathematics are often hidden and rarely critiqued leaving power with those who develop, understand and apply mathematical models.

Reflection, critique and action

Reflection, critique and taking action are connected themes across the various constructions of critical mathematics education. Arlø and Skovsmose (2002) highlight the relationship between reflection and action and include reflection as a key component of landscapes of investigation. They describe three different aspects of reflection; what can be addressed by reflection, who is carrying out the reflection and the context in which the reflection is carried out (Arlø and Skovsmose 2002, p. 184). Mathemacy has reflective knowing as one its three competencies with the task of being a catalyst for critical awareness and this could lead to criticising systems established by means of mathematics (Skovsmose, 1994, p. 125). Reflective knowing enables one to identify the nature of political and economic understandings that direct the application of mathematics and enables one to question the choice of mathematics, the accuracy with which it is being applied, the reliability of the results and how the application of mathematics relates to broader contexts.

STUDENT AND COURSE CONTEXT

Critical mathematics education is context bound (Skovsmose, 2005, p. 3). For example, Vithal (2003) writes from the context of post apartheid South Africa, Frankenstein (1998, 2000) writes from the context of marginalised, urban adults in the United States, Knijnik (1998, 2002) writes from the context of Brazilian landless people while Skovsmose (1994) writes from a Danish context. Given this, I will briefly describe key characteristics of the students enrolled in the two integrated mathematics and science education courses and the courses themselves.

The Primary and Middle Years program commenced in 2005 and develops students as generalist years 3 to 9 teachers with two areas of curriculum specialisation for teaching in the middle years (years 6 to 9). The program and courses within it are constructed around a set of seven guiding principles.[1] Two of these principles, the principle concerned with equity and social justice and the principle concerned with sustainability reflect the general aim of critical mathematics education.

Key student characteristics

Two student characteristics in particular impact on the structure and learning content of the two integrated courses. Firstly, the diversity of students enrolled in the program is not representative of the wide cultural and soci-economic diversity found in South Australian schools. A challenge for the program is to develop students' understanding and appreciation of the diverse range of cultures, values and socio-economic situations within school communities as well as prepare them to confidently and competently cater for the breadth of social, emotional and academic needs of the students they will teach in their future classrooms. Secondly, the vast majority of students in the program have a narrow view of mathematics and lack confidence to "do" and learn mathematics. This impacts on their self image as early career generalist teachers of mathematics (Paige, Chartres & Rowell, 2004). Most believe there is 'one mathematics,' dominated by number, rules, techniques and correct answers. The ideas of a mathematics education for democracy and citizenship and that mathematics is not neutral are foreign to students.

The structure of mathematics and science education courses

The two courses integrate mathematics and science education to maximise the opportunity to engage students with social justice principles and ethical concerns and to connect these to students' life-worlds. Integration has two meanings with respect to the courses. The first meaning refers to learning topics that integrate aspects of mathematics and science. The second meaning refers to integrating pedagogy and research drawn from both mathematics and science education. Key characteristics of the integrated courses include a workshop structure that employs interactive practices, students working collaboratively and authentic assessment tasks. Learning topics in each course may employ a solely mathematics vehicle, e.g. chance, a solely science vehicle, e.g. invertebrates, employ an integrated focus such as sorting and classifying or focus on an interdisciplinary issue. The notion of critical numeracy (Johnston 1994) and arguments for a socio-political science education (Hodson 2003) informed the teaching team's early ideas about implementing a critical education focus, particularly with ecological and social sustainability in mind (Chartres, Lloyd & Paige, 2003; Paige, Lloyd & Chartres, 2005).

The main aim of the two courses is to develop students' understanding of curriculum and their ability to teach mathematics and science and to plan for, resource and assess years 3 to 9 student learning. While an interdisciplinary approach with a critical mathematics and science focus enriches the courses it places tension on prioritising outcomes. This tension is visible when students undertake their practicums or school placements. The tension between aims, experiences and teaching approaches that reflect a critical mathematics education and those of a more traditional approach to mathematics is well documented by Skovsmose (1994) and Vithal (2003).

CRITICAL MATHEMATICS EDUCATION AND A LEARNING TOPIC

'Data and living sustainability' and 'A place in time' are two learning topics that have a critical mathematics education focus. Both engage students with the issue of sustainability. Through the topic 'Data and living sustainability' students identify their personal actions in living sustainably and then focus on water use from a personal and a global perspective. Currently South Australia is in drought and communities are implementing stringent water restrictions. One outcome for this learning topic sees students plan, undertake and critique strategic action to decrease their water footprint. The topic 'Data and living sustainability' has much to offer this paper but the context of working with data is well documented by Frankenstein (1998, 2001) and the context of resources (energy) is used as an example of project work by Skovsmose (1994). As such the discussion about how students engage with critical mathematics education is drawn from the learning topic 'A place in time.' My analysis of 'A place in time' identifies several aspects that support students to engage with critical mathematics education and two aspects that need further development.

'A place in time' was first developed from the need for students to familiarise themselves with their new university environment. It also presented an opportunity to engage students with a learning topic that modelled an interdisciplinary approach and introduce them to a critical perspective for mathematics and science education. The program, courses and campus were new to both students and academic staff when the topic was first taught. 'A place in time' has since proved to be a powerful learning topic for subsequent years' students. Two strengths of this learning topic with respect to critical mathematics education are the real-life context (Ernest, 2001; Skovsmose, 1994) and deep, complex questions (Frankenstein, 1998) that situate student learning. Students focus their investigation of ecological sustainability on a campus that is undergoing continuous redevelopment and provides opportunities for students to experience the impact of human activity first hand. Deep questions, “Where are we?” “Who and what was here, is here, maybe here in the future?” “What is the impact of human activity?” “Which aspects of the physical and biological environments would you prefer to be here in fifty years time and what action is required for this to happen?” frame the students’ investigations.

A further strength of ‘A Place in time’ with respect to critical mathematics (and science) education is its interdisciplinary approach. Student pairs begin by exploring many campus locations with respect to the built and natural environment and from this choose a mature tree and its surrounds as their adopted space – their ‘place in time.’ Over three, three-hour workshops and out of class tasks, students use a mathematics lens, a science lens and an environmental lens to investigate aspects of the physical, biological, and socio-political dimensions of their adopted space. The mathematics lens supports students to use aspects of measurement, pattern and data handling to explore the location, size and characteristics of the physical, biological and social environments. The science lens supports students to investigate the surrounding physical environment through weather and soils and to investigate the biological environment through plant and animal structure, habitat and plant–animal relationships. The environmental lens introduces students to the ideas of future scenarios and sensory experiences as a means to explore change over time and the impact of human activity as part of this change. Here, interdisciplinary means using each lens to build a connected understanding of sustainability and explore key concepts that underpin sustainability including, interdependence, intergenerational and intragenerational fairness, equity within and between species, stewardship and ethical action.

‘A place in time’ reflects many characteristics of a landscape of investigation including, working collaboratively, posing questions, finding ways to answer these questions and building mathematical understanding as part of the investigation. Many student questions early in the topic are typically ‘how’ and ‘what if’ questions (Skovsmose, 1994, 2001a) where they negotiate and make decisions about the mathematics they apply and how they apply it and about the direction of their investigations. For example, questions like, “How can we measure the height and size of canopy of our tree?” “Which is the best kind of map to represent our place?” “How can we show directions on an aural map?” “What is the best way to sample who or what visits our space?” “How can we sort and represent the natural and manufactured items found in our space?” see students coming to grips with familiar and unfamiliar measuring strategies, making decisions about data they collect and how to work with this data. In some ways this approach places student democratic action as part of directing the learning experience (Skovsmose, 1994). Part of my role is to encourage students to share and decode their popular knowledge before inquiring about the academic mathematics to solve a problem. This strategy is more successful for students who are prepared to take risks. While this strategy is sometimes sabotaged by students' lack of confidence with mathematics and their view of one correct way, it does highlight that learning mathematics can be developed through an investigation (Skovsmose, 2001a). The approach also reflects Frankenstein's goal of understanding the mathematics and, in part, Knijnik's ethnomathematics approach but falls short of focussing on the power relations between the students' popular mathematics and the academic mathematics (Knijnik, 1998) or engaging with mathematics of political knowledge (Frankenstein, 1998).

The role of mathematics in a socio-political context is the most problematic aspect of 'A Place in time' with respect to critical mathematics education. Students apply mathematics to investigate their space and identify issues around human impact but they only superficially explore the socio-political milieu that controls their space. For example, the futures scenario asks students to consider what their space was like in the past and the impact of past and current human activity on their space. It also asks them to use their investigations to frame a preferred future for their space including what will be there and how it is used from an ecological sustainable perspective. Finally, the futures scenario asks students to describe the actions required to realise their preferred future. This is not extended to actually taking action where students would encounter the socio-political milieu more explicitly. That is, the topic does not explicitly engage students with the mathematics of political knowledge (Frankenstein, 1998) or with mathematics in action (Skovsmose, 2001b). Incorporating opportunities for students to investigate and critique how mathematics is used to plan for ecologically sustainable development on campus and critique the underlying economic and political interests would strengthen the critical mathematics education focus. So too would opportunities for students to participate in the planning process, share the results of their investigations and argue their views.

Students' reflections and critiques are focussed on their use of mathematics. For example, students reflect on and critique the appropriateness and reliability of the mathematics and strategies they choose over the three workshops. At times, they critique and provide feedback about the strategies and findings of their peers. This often happens informally during a debriefing period and more formally when students use a draft another pair's summative findings to locate and explore this pair's place in time. Reflection and critique is also a key part of an assessment task. The assessment task has two parts. Firstly, student pairs produce a pamphlet using each lens to describe their adopted space, its inhabitants and visitors, the impact of human activity, and a futures scenario. Secondly, each student produces an overview of how his/her investigation has impacted on personal perceptions of the adopted space, raised issues about sustainability and informed the suggested actions to preserve the adopted space. Finally, students describe what they consider to be the strengths of implementing a topic like 'A place in time' in a primary or middle years classroom and the challenges they may encounter when doing so. Broadening the topic to engage students with the socio-political milieu as described above may also provide opportunities to shift student critique towards using mathematics to critique the plans and actions of others rather than solely critique their application of mathematics.

CONCLUSION

Are my students engaged with critical mathematics education? The learning topics ‘A place in time’ and ‘Data and sustainability’ challenge students to consider the notion of critical mathematics education, a mathematics education that can empower and leads to citizenship and democratic participation. Such learning topics engage students with real issues drawn from their life worlds in a way that suggests the application of mathematics is not neutral and supports them to experience how their application of mathematics informs their decisions and actions. Having said this, ‘A place in time’ has some way to go to truly engage students with the socio-political milieu that is the landscape of critical mathematics education.

In closing, both learning topics draw on the idea of exemplarity (Skovsmose, 1994, p. 75). From a citizenship perspective, students engage with issues drawn from local communities that are examples for future investigations of similar issues. From the teaching perspective, students have participated in learning topics they can reflect on, adapt, refocus and implement as aspects of critical mathematics education in their future years 3 to 9 classrooms. It is my hope they do so.

NOTES

1. The University of South Australia’s Primary and Middle Years teacher education program is structured on seven principles. Namely, social justice and equity, futures thinking, sustainability (Education for one world), education for community living (Placed-Based Education), well being and relationships development, professional competence, and program and course delivery that reflects and models the first six principles.

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