MULTICULTURALISM, HISTORY OF MATHEMATICS AND SCHOOLBOOK OF THE THIRD CLASS IN PRIMARY SCHOOL IN GREECE

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During the last decades the movement that supports the cultural and historical dimension for the learning and teaching of mathematics has set on the stage of the international level the necessity of using elements from the global and local history of mathematical concepts. The communities of teachers of mathematics of the Secondary education but also some groups of Primary school teachers agree with the new possibilities that the introduction of the above dimensions in the teaching of mathematics offers. The big problem that does not permit them to often use all these possibilities is the curriculum and the schoolbooks they use, which demand the development of capacities of techniques for doing the operations and the algorithms that do not permit teachers and consequently students the substantial understanding of mathematical concepts. The fact is that the mathematical education, the books, the training of teachers does not cultivate such approaches. In the core of all of the parameters mentioned above is an extreme formalism. We believe that the introduction at the undergraduate level in the Universities, in the context of the pedagogical Institutes of Training of teachers, in the curricula and in the school textbooks of all classes could change the approach of the teaching of mathematical concepts.

In our presentation we are going to present one example of introduction of elements from the history of mathematics in the context of the school book and of the teaching of mathematics for the third class of Primary education in Greece. Our intervention contains examples from geometry and arithmetic.

The introduction of cultural and historical elements for the teaching of mathematics is a fact for many educational systems all over the world. Our main objective is to exchange with other colleagues from other countries the best practices of all these efforts and gradually develop a kind of paradigm of the best practices which could positively influence the future decisions.

INTRODUCTION

During the last decades many people working in the field of mathematics education accept the thesis that it is very important to show first of all to the teachers of the subject and consequently to their students the way that mathematical thinking and the different applications developed in different civilizations and in different historical periods. This could be another instrument for the teaching and could permit a wider understanding of the concepts embodied in mathematics. The International Congress of History of Science in Mexico City, in 2001 was under the theme of the Cultural

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Diversity, and the main characteristic of this Congress was that the History that shows the diversity, rather than the universality of mathematical and scientific development adds a very exciting dimension in the subject because saying that Mathematics has a history can change the image this science has in the eyes of teachers. We can not summarize mathematics as a final product but as a procedure in time and in space. The use of historical elements of a notion or a technique can offer to teachers and also to their students two important occasions for working in two dimensions (Cerquetti et al., 1997).

First, working in contexts of different historical time where they leave the status of a stereotype knowledge which has come from the skies and put in its place a knowledge constructed by the humanity: mathematical notions were not always here but they have been constructed and changed during history for resolving problems and being instruments of response to important questions. By saying that mathematics has a history you give a constructivist image, which goes against the dogmatic impressions for this science.

Second, working in different geographical and cultural areas, because the history of mathematics permits to re-establish the human construction in the context of the different cultures and civilizations: mathematics is Babylonian, Egyptian, Greek and Arabic, Hindu and Chinese, Mayan and Incas, that of the middle ages, of Renaissance or that of the 17^{th} – 18^{th} century. Saying that mathematics has a history permits a multicultural approach to this science.

The history that interests teachers is epistemological, conceptual and historico-cultural and before coming to be a teaching instrument for this subject it should pass through the training of teachers of Primary and Secondary Education. The purposes of this introduction in the training and teaching of mathematics are various and are fixed more or less in the following (Fauvel & Van Maanen, 2000):

- to humanize mathematics,
- to put mathematical knowledge in the context of a culture,
- to give to students the opportunity to change their beliefs for the subject,
- to find and analyze epistemological obstacles and notions that are not very well understood by the teachers and consequently by the students,
- to show that mathematics has a history and was influenced by cultural and social parameters,
- to be another interdisciplinary project that could be studied with the students,
- to develop and enrich mathematical knowledge included in the curriculum.

The ways and strategies used for this introduction in the training and teaching of the subject are: histories, construction of activities, construction of exercises, reproduction of manuscripts, portraits, biographies, interdisciplinary projects, use of primary sources, use of new technologies etc. (Barbin, 1987).

The main characteristics of this kind of intervention in mathematics curricula, training of teachers, textbooks etc. could be a way of working against an ethnocentric view and an entry into different cultures by valuing the different styles and branches of the mathematical activity.

The broadening of perspectives, that the historical study could give us could also give a new impetus to teachers and to their students to search their own background and culture as well as within the cultures of others and (hope) try to understand that what is found is a part of a global heritage rather than merely a national or regional one.

Mathematics is text, artefacts, inscriptions, instruments, books and technical devices that have been developed in different places in different historical periods for particular reasons and the understanding of these reasons can help all of us to relate mathematical ideas to something more global than simply their own immediate environment (Fauvel & Van Maanen, 2000). Integrating elements from the history of mathematics of other cultures and traditions in the schoolbooks of the discipline in an operative manner is one of the major tasks of the mathematical community because in this way we offer to all students the possibility to study all these. In the following we will discuss our effort to integrate historical elements and methods in the schoolbook of the third class of primary school in Greece.

The methods proposed should be in an operative way integrated in the text and help the student to do an operation and to understand a mathematical concept. Our history is not personality based but problem based, which is an important parameter of what we call “Mathematics of Nature and Life”. Some of the basic principles of this approach about the teaching of mathematics are the following: it is very important and special attention shall be given to the content of the didactical situations and problems used for the introduction and application of mathematical concepts. Contents and situations must be pleasant and come from the pupils’ world and from their preexistent knowledge. They are interested in reporting nature, culture and the history of mathematics (Lemonidis, 2005).

THE CULTURAL AND HISTORICAL ELEMENTS FROM THE SCHOOL BOOK OF THE THIRD CLASS (8-9 YEARS OLD) OF PRIMARY SCHOOL IN GREECE

In Greece, all students use the same schoolbook for mathematics and all other subjects offer every year by the Ministry of Education and under the responsibility of the Pedagogical Institute.

In the following we are going to present paradigms from the schoolbook, the students’ exercise books and the book for the teacher (Lemonidis et al., 2006 a, b).

Tangram

In chapter 3 we can see the introduction of Tangram. In this example students observe and find the same figures in different positions on the page level. Via this they are coming to recognize congruent figures in the context of the Chinese mathematical culture. In chapter 42 they can construct the tangram with a carton patron, and they can exercise themselves and play.

The context in which Tangram is introduced is the study of geometrical figures and solids. The objectives and the didactical instructions are that pupils already know the names of the basic geometrical figures and solids. The objective of this intervention is to re-establish the main characteristics of these figures and to reinforce their knowledge. The main goal is that the pupils at the end of this procedure should recognise and isolate a figure from a synthesis of geometrical figures. Another objective is to make pupils to recognize figures in different positions and not only in the prototype position. It is very important for students to discover empirically the relations between the figures. In this direction we can develop activities that permit to use known figures for the production of new ones.

In the exercise book in the activity 2 pupils construct two paper triangles and regard them as two pieces of tangram. They need to put them together and to figure other schemes, the square and the isosceles triangle.

After almost 40 chapters in the context of the study of Puzzles, pavement and mosaics (ch. 42) we use once again the Tangram for approaching the application of optical procedures of analysis and synthesis of geometrical figures. All these activities develop the optical geometrical capacities which are extremely useful. They introduce pupils to the concept of surface to a pre-area study situation and go further to study the properties of the figures.

In this context we can make reference to China civilization and its mathematical tradition.

Multiplication and Division

In the chapter of multiplication and division (ch. 6) we introduce the Pythagorean table. The pupils discover how to read in the Pythagorean table that contains multiplies of numbers by using the lines and columns. We ask them to find some multiplications. They observe the evolution of the multiplications by 1, 2, 5 and 10. We ask them to find how many times and where we can see, for example, the number 24 and to write down the different multiplications.

We give elements on Pythagoras life and activities and on the different kinds of numbers he developed: pairs, impairs, square numbers, triangular numbers and of course his very well known theorem. We propose the use of an interdisciplinary activity by using the field of history and geography.

Numbers to 3.000

In chapter 14, we are working on numbers to 3.000 by introducing the Roman arithmetical system. We are showing to pupils that except the symbols of monads (1, 10, 100 and 1000) there are symbols for the numbers (5, 50 and 500). This is an evolution which does not permit the repetition of symbols like other ancient arithmetical systems. The Roman system goes beyond M (1000). By confronting it with the Hindo-Arabic origin numerals in use today we found that it has basic disadvantages: it has a lot of symbols, repetition of the same symbol and non existence of number zero.

Introduction of the "Greek multiplication" in teaching

In the chapter 29, we continue the practice of pupils on multiplication with the aid of squared paper. We guide them to pass from the figure of the squared paper, where the factors of the product are separated by 10, in the corresponding table of multiplication. This kind of multiplication is called "Greek multiplication". It was presented for by Eutocius - a Greek scholiast of 6^{th} century A.C. (Mugler, 1972). The Greek multiplication is a good way to introduce pupils to the algorithm of multiplication in use today.

In pupils' book in the first activity students work in groups and we ask them to cut and construct on the squared paper a rectangle with 24 x 35 squares. When they cut the rectangle we ask them to find how many squares they have in the rectangle. Students give different solutions and we advise them to trace the rectangle, and to figure squares and rectangles and by this way to find the number of squares. By discussing, the teacher, show them how to trace and to separate their rectangle with the way shown in the book: to establish a table with 20 and 4 on the two lines and 30 and 5 on the two columns. We ask them to compare this table with one which could be constructed with the logic of the previous course (a table with 10, 10 and 4 on the lines and 10, 10, 10 and 5 on the columns). At the end we introduce the way by which we can multiply two two-digit numbers by the "Greek multiplication".

The objective of the activity 3 is to make pupils practice in using the table, to analyze two digit numbers and to calculate the products.

In the exercise book we propose the multiplication of two digit number with one digit number and we ask in activity 2 to complete the products in the table. The objective is to observe that two products represented side by side result by multiplications in two cells of the table. In the following activity (no. 3) we have a multiplication of two digit and three digit numbers and the objective is to make them practice by analyzing numbers and use the distribution property of multiplication to addition.

In the following chapter we introduce the algorithm of multiplication and one of the goals is to interpret the products of the algorithm that come from the multiplication of the multiplier with the multiplicand by using as a base the multiplication table and to consider them as the shorter expression of the products of the multiplication table.

"Greek multiplication" is very useful to students by using a table for analyzing the multiplicand and the multiplier. The two factors of the multiplication are analyzed in units, tenths, hundreds etc. in the columns of the table. We put the analyses of the multiplicand in the columns and of the multiplier in the lines. We propose to put them in these places because there is a correspondence with the classical and typical algorithm in which we put the big number – multiplicand- up and the small – multiplier-down and multiply the down number with the up one.

Eutocius used to multiply by a diminishing series. He multiplied the biggest numbers and afterwards the smaller ones. In the typical algorithm multiplication must be done from right to left, the digits were put by increased series.

In this book the “Greek multiplication” is used as an introductory phase for the introduction of the typical algorithm used today. In more details in the first phase we give to pupils problems for multiplying multidigit numbers to develop atypical methods for the calculation. The “Greek multiplication” is introduced via a geometrical context, with situations of measurement of surfaces in a squared paper. Pupils measure surfaces of squares and rectangles by a small square as unit. For the most quickly and easiest calculation of the surfaces they use the table of multiplication. Via this way they use the “Greek multiplication” with the help of the table. By the use of the “Greek multiplication” we can give explanations to the typical algorithm. The goal is that pupils can understand the way of production of the products and the place-value of numerals of the factors of multiplication.

The above examples are included to encourage teachers to find different methods from history for doing calculations which have applications that are both modern and appropriate.

Decimal fractions and decimal numbers

In chapter 35 we have the decimal fractions and decimal numbers, where we make a reference to the historical appearance of decimal numbers by writing that fractions were known from Antiquity. They were used by Egyptians in the 2nd millennium B.C. In Europe mathematicians used them in contrast with the decimal numbers. Al-Kashi, an Arabic origin mathematician (died on 1429 A.C.) was the first who introduced the theory of decimal fractions and established that operations can be done by the same way as integers. We have to wait until the 16th century, when Simon Stevin (1548-1620) a Flemish mathematician introduced the way of writing the decimal numbers with the following way: 5,237 were written 5 0 2 1 3 2 7 3.

He has noticed that this number is equivalent with 5 + 2/10 + 3/100 + 7/1000 or 5.237/1000.

Numbers to 7.000

In chapter 40, for approaching numbers to 7.000, we use the arithmetical-alphabetical number system from ancient Greece

In the context of pupil’s book, in activity 1, we have the introduction of the ancient Greek arithmetical-alphabetical system. This is not an arithmetical system with place – value numerals. In this system they have not a notion or symbol of zero. The goal is not to learn this system but to experiment with it and to use it, by transforming numbers from one system to another. Pupils are invited also to make comparisons with the Roman arithmetical system.

Motifs

In chapter 48 we introduce and practice arithmetical and geometrical motives and especially in activity 3 we use Pascal's triangle which is a very well known arithmetical triangle and has been used to resolve problems on combinatorics and probabilities.

INSTEAD OF EPILOGUE

These examples help us to see the different aspects of mathematics and its development can be discussed when a problem-based approach is enhanced by adding a multicultural dimension to the teaching of mathematics. These could help students to see ways in which differences in historical periods, geographic location, culture and beliefs have influenced developments in mathematics (Fauvel & Van Maanen, 2000). In turn, they may help students to understand better the concepts of operations such as multiplication etc. This approach allows students and teachers to think of mathematics as a discipline of continuous reflection and action influenced by thoughtfulness, reasoning, known procedures, intuitiveness, experimentation and application of practical situations as is always the teaching context of the subject in the Primary school education.

In the teaching of mathematics in the third class of Primary school in Greece there are opportunities for introducing aspects of the history of mathematics through examples (Roman arithmetical system, ancient Greek alphabetical number system, Greek multiplication, tangram, Pascal's triangle, decimal fractions and decimal numbers, Pythagorean table) from different cultural perspectives (Nikolantonakis, 2005). Pupils and teachers are able to see how one culture or one group of people, or one geographical area has influenced another or added to understanding already gained in a different setting. They can see that mathematics is a human enterprise and not a constantly upward movement towards perfection.

Studying and understanding the methods that other groups of people have developed in response to their needs may well help students to identify the particular characteristics of the method being taught to them and thus better understand a particular concept. It opens up the possibilities of comparisons and the recognition of diversity and enables us to see that an exchange of ideas can be made from the security of a mutual concern to explore mathematical concepts in the environment of the people who use it.

Multiculturalism is the identification and celebration of diversity, the respecting and valuing of the work of others, the recognition of different contexts, needs and purposes, and the realization that each society makes and has made important contributions to the body of knowledge that we call mathematics. Given this view, the inclusion of a multicultural dimension in our teaching of mathematics makes a significant contribution to humanist and democratic traditions in mathematics education.

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