A PLURALISTIC VIEW OF CRITICAL MATHEMATICS

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What counts as critical mathematics (CM)?What mathematical and political messages characterize CM and how explicit do they have to be? How do the social context of instruction and the marginal positioning of students figure into whether a pedagogical practice is CM and politically engaging? We argue a pluralistic conception of CM. We posit that what counts as CM depends as much as on the students—their sociopolitical and institutional positions and their goals—as the explicitly political nature of the curriculum. Our discussion points to problems with what we believe are current, overly narrow theorizations of CM.

In this paper, we outline a perspective of critical mathematics that admits varied forms, each characterized by attention both to mathematics learning and teaching and to social critique, contributing to movements for greater social justice. What varies among the forms is the balance between mathematics and critique or what is highlighted and what is implicit. Further, we contend that this perspective implies a pluralistic, nuanced yet political, view of what constitutes critical mathematics.

To illustrate our perspective, we present and analyze two sets of data that instantiate distinctive forms of critical mathematics. These data come from classroom-based investigations where we independently taught and collected data. The first data set comes from an entry-level, college mathematics course with students from historically marginalized communities of color. Traditional measures of mathematical achievement positioned these students as mathematically "deficient" and even incapable of pursuing mathematics beyond college-level algebra. While younger, the urban high school students in the second context were not dissimilar from those in the college context. All had failed a geometry course previously and most did poorly on a pre-examination. Being students of color from communities of lower socio-economic status, many in both settings had internalized hegemonic and academically destructive messages about themselves.

Based on our presentation and analyses of these data, we will discuss the extent to which the instructional settings from which they emerged evidence critical mathematics and elaborate on how they represent social critique in the service of social justice. To distinguish the two approaches to critical mathematics that our data suggests, we consider the first approach CM1 and the second approach CM2.

CM1: ACCESS TO ACADEMIC MATHEMATICS AND THE IMPLICIT POLITICAL MESSAGE THAT CARRIES

An objective of critical mathematics ought to be to engage students, socially marginalized in their societies, in cognitively demanding mathematics in ways that help them succeed in learning that which dominant ideology and schooling practices

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position them to believe they are incapable. Such opportunities for learning have been integral to struggles of socially excluded sectors (D'Ambrosio, 2001; Gerdes, 1997; Moses & Cobb, 2001).

In 2007, the people of the US celebrated the fiftieth anniversary of the landmark racial integration of Central High School in Little Rock, Arkansas. In September of 1957, defying a federal court order, armed National Guardsmen along with an angry, racist mob blocked the precedent-setting path of nine, 15-year-old African American students who attempted to attend the all European American high school. After a three-week standoff, President Dwight D. Eisenhower, embarrassed by the negative coverage in the world press, ordered the US Army forces to escort the black teenagers to school and put down the mob. These teenagers continued to suffer a litany of verbal and physical assaults. Why did they endure the suffering? Melba Patillo Beals explained precisely what integration meant to her and the other eight students: "These people had language labs. They had typewriters.". Furthermore, she says, "We didn't go to Central to sit beside white people, as if they had some magic dust or something. I would not risk my life to sit next to white people. No, no, no, no. We went to Central for *opportunity*".[1]

Forty years after the Little Rock events, racial integration of schools was overshadowed by a powerful counter-imagery. The entering class at the University of Texas Law School contained only two non-white students. This was applauded by Lino A. Graglia, the A. Dalton Cross Professor of Law at the University of Texas, who observed that African American and Latino students "are not academically competitive with whites" and that they belonged to a "culture that seems not to encourage achievement. Failure is not looked upon with disgrace." Notions that people of color are culturally or genetically academically incapable permeate dominant discourses, both popular and learned (see, for instance, Herrnstein & Murray, 1994). At the end of 2007, the world-renowned geneticist and Nobel Laureate, James Watson opined that blacks are less intelligent than whites[2]. The struggle against such dominant-ideological narratives and for learning opportunities continues.

A challenge for critical mathematics educators is to counter hegemonic narratives about who can do mathematics and to reconstruct the role of mathematics in the struggle to empower learners whose mathematical powers have been underdeveloped. This challenge demands a critical mathematics curriculum that inverts traditional pedagogical structures of instruction so that the teaching of mathematics takes second place to the development of students' mathematical ideas, heuristics, and reasoning. This inversion is what Gattegno (1987) calls "the subordination of teaching to learning."

This pedagogical notion and the historic struggle for learning opportunities inform the course from which the CM1 data come. The course —pre-precalculus— was designed for students who entered the university underprepared for university-level courses in mathematics. The students were African Americans and Latinos from economically impoverished school communities in close proximity to the university campus. The university considered these students as “at-risk” of not succeeding academically and required that they attend a six-week summer program designed to equip them with study and other academic skills that would help them succeed in their university studies. The pedagogy of this course was trangressive in that students participated in challenging, cognitively demanding mathematics[3] —mathematics that had previously been “off limits.” Doing serious mathematics allowed students to challenge dominant ideological messages they had internalized about their own ability —and the ability of people like them— to do academic mathematics.

The following investigation from a unit on functions is typical of the course’s mathematical demand and attempt to challenge students’ internalized hegemonic sense of themselves as mathematics learners. In this investigation with graphing calculators, students were presented with mathematical rather than contextual situations to explore. They later related their investigations to contextual situations. In this unit, they worked in small groups to investigate several families of curves of single-variable functions. They had to distinguish among the independent variable, constants, and parameters to know what to vary in function such as $l(x) = e^{-kx}$. Other functions they investigated include these:

a) $f(x) = ax^n + b$

b) $g(x) = \frac{a}{x^n} + b$

c) $h(x) = a \sin x$

d) $j(x) = \cos(wx + d)$

By combining these functions and adding parameters, students crafted new functions and constructed function expressions to correspond to given curves. For example, students attempted to determine the symbolic algebraic description of a function whose graph corresponds to the shape of the curve depicted below in Figure 1.

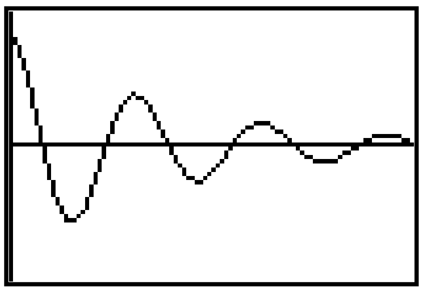

Figure 1. A curve that represents damped harmonic oscillation

Afterward, students related their function expression to the physical context of a damped harmonic oscillator such as a mass at the end of a spring (a vibrating spring) or a mass at the end of a cord (a pendulum). Finally, they explored questions such as the ones below aimed to increase their awareness of relationships between the curve, function expression, and movement of a pendulum affected by friction:

1. For the function $f(t) = a\mathrm{e}^{-kt}\cos(bt + c)$, where f(t) is the displacement (in inches) of a mass from its rest position, t is the elapsed time (in seconds), and a, k, b, and c are parameters, determine parameters values that give a good fit to the following data:

Time	Displacement
0	10
4.5	9.2
9.4	8.9
13.9	8.3

2. At what time does the pendulum first pass its resting position?

3. At that time, what is the speed of the pendulum?

At the end of the investigation, students wrote reports, summarizing their findings. We discuss and present the excerpts from one student's report. This student, Rohan, as a condition of his acceptance to the university, attended the six-week summer program because the university considered him an "at-risk" student because of his educational background, his intended participation in the university's football (soccer) program and his weekend free-lance job as a disc jockey (DJ).

In his report, Rohan first states a conclusion of his investigation: that the curve in Figure 1 "represents the distance a pendulum travels over time." He then declares that his goal is "to find a function whose graph resembles" the curve and begins his exposition by reviewing his knowledge of the functions he believes are involved in the curve, namely some form of $\cos x$ and e^x. He next provides illustrations of their graphs and discusses each of these functions.

Concerning the cosine function, he shows in his report the effect of multiplying the independent variable by a constant, $\cos bx$. After illustrating how different values of the parameter *b* affect the period of the cosine curve, he explores the effects of multiplying the dependent variable —$\cos bx$— by a constant, *a*, and explains that the value of the parameter *a* affects the amplitude of the cosine curve. Reflecting on the graph of the curve in Figure 1, he notices that it tends to be a dampening, oscillating function of *x* and suggests that the graph results when the cosine function is multiplied by some decreasing function.

After some thought, the decreasing function of *x* that Rohan chooses to dampen the $\cos bx$ curve with is e^{-kx}, assuming $k > 0$. He notes that e^{-kx} decreases as *x* increases and that as the value of *k* increases the graph of e^{-kx} "decreases proportionally slower and approaches the *x*-axis." He then speculates that "[s]ince e^{-kx} is a decreasing function whose graph decreases slowly along the *x*-axis as the value for '*k*' gets smaller, and the graph of the distance a pendulum travels over time is a decreasing cosine graph along the x-axis, I modify the function $f(x) = a\cos bx$ to $f(x) = ae^{-kx}\cos bx$." In earlier sections of his report, he demonstrated his awareness of the effects that the parameters a, b, and k exert on the behavior of his modified $f(x)$. With a suitable choice of parameter values and with a restriction of the domain of $f(x)$ to $0 \leq x \leq 15$,[4] he states his discovery that the graph of the function $f(x) = 5e^{-\frac{1}{32}x}\cos 4x$ resembles the one given in Figure 1.

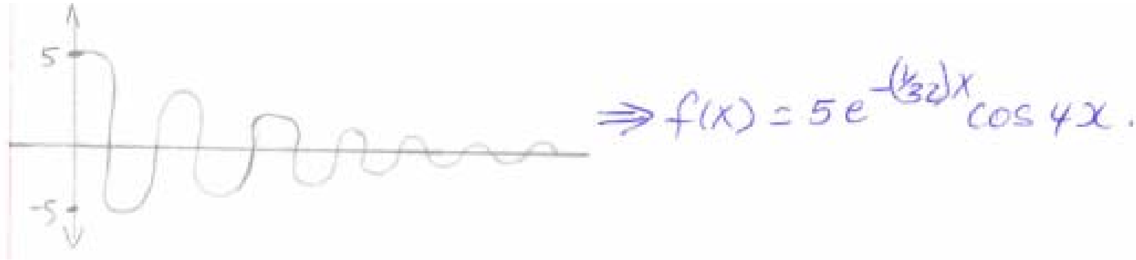

Figure 2. Rohan's graphical and algebraic representation of the curve given in Fig. 1

In the investigation, Rohan and his peers worked as mathematicians, generating knowledge that was new to them. His description of his discovery resembles the writing of mathematicians. It follows a rather linear, logical order. He carefully states is conclusion, then discusses what he knows, and illustrates his knowledge appropriately with both algebraic and corresponding graphical examples. Characteristic of writings in textbooks and professional journals, he reveals neither the affective states he experienced during his struggle nor the messy, perhaps even non-methodical, scratch-pad work in which he engaged. Those affective and messy experiences remain private.

This raises an important instructional point. It is valuable to point out to students the contrast between the messy affective and cognitive experiences of their work and the emotionless, logical exposition of their laboratory reports. This may serve to demystify mathematics textbooks and to make students cognizant that the struggle of discovery is likewise usually missing from the narratives they read in mathematics texts.

Among other issues, this example illustrates the power of a graphing calculator to allow the study of functions and physical situations that educators traditionally consider beyond the mathematical reach of pre-precalculus students. Typically, students in advanced calculus study this function as the solution to a problem in physics that, applying Hooke's Law and Newton's Second Law, leads to a second-order linear differential equation that models the movement of a vibrating spring or a swinging pendulum affected by friction.

Just as important, this report exemplifies how graphing calculator and writing are tools for educating one's awareness. By experimenting and focusing his attention, Rohan informed his awareness of the effects of varying the values of parameters of $f(x) = ae^{-kx}\cos bx$ and the corresponding graphical changes. Aided by a graphing calculator, he constructed a function whose graph resembles the one in Figure 1. Writing the report prompted him to articulate connections between information embodied in algebraic and graphical representations of functions. The connections he expressed are displays of his mathematical awareness. Moreover, to inform and persuade his audience, writing obligated him to arrange his awarenesses in coherent, logical order that possibly led him to other insights.

Pedagogical and ideological issues arise from examining both the content and process of CM1. Pedagogically, the course combined the use of graphing calculators and transactional writing to promote dialogue and reflection with challenging, cognitively demanding mathematical situations. These were crucial for engaging students to focus their attention and become aware of mathematical features of functions. The activities of the course engaged students in examining functions in context as well as context in functions. It challenged their internalized beliefs about themselves as students. The pedagogy that informed this work indicates an ideological position concerning the education of students' awareness in contextual and mathematical situations. This pedagogy responds to traditional instructional practices about which one can ask the following question: What is the view of students of color that positions lecture and rote practice as the central mode of instruction and that holds students hostage to the minutia of textbook discourse?

CM2: CRITICAL MATHEMATICS WITH AN EXPLICIT POLITICAL MESSAGE

The second example of critical mathematics, what we label CM2, is more explicitly political and, less academically focused, than the CM1 example. This CM2 example comes from a geometry course taught by the second author (Brantlinger, 2007). The course took place in a night program at an urban high school that served lower SES students. All but one of the 27 night course students was African American or Latino and all were making up credit for past failure in geometry. Many were pushed into the night school program for various "offenses" including truancy, gang involvement, and teen pregnancy. In this lesson, the instructor asked students to interpret a relatively complicated statistical chart that summarized data on the distribution of recess by the racial make-up of public elementary schools in Chicago in 1999. As Figure 3 shows, there was an inverse relationship between race and recess: as the student-of-color population increased the amount of recess time decreased precipitously.

In contrast to student engagement in standards-based reform activities that comprised four-fifths of the night course curriculum, the politicized and racialized CM2 activity appeared to engage several previously disengaged students. That is, the "Race and Recess" activity resonated with some—though not all—of the night school students in ways the "apolitical" reform mathematics activities had not resonated to date.

At the same time, there was considerable mathematical confusion surrounding the chart. While it was clear that students had little past experience with data analysis of this type, the videotaped data suggests there was a tension between students' firsthand knowledge of segregated schooling and the existence of "whiter" public schools in Chicago reported in the chart. This tension appears to have contributed to problems students had interpreting the chart. For instance, Osvaldo first read and then evaluated the chart stating, "percent white students, it throws it off." Shortly thereafter, when asked to provide a mathematical interpretation, rather than directly referencing the chart, he drew on his own perceptions of racialized socioeconomic distinctions, stating, "the white people have better jobs and stuff —they live in a better community and so they can afford to go to [schools with recess]." Later in the activity, Osvaldo asked the instructor if schools that were majority white actually existed in Chicago. While it could be that he was more interested in discussing the political than the mathematical, there appeared to be a disjuncture between Osvaldo's past experiences in schools with small white populations and the real world data that documented the existence of whiter schools in Chicago. Osvaldo was not alone in his confusion. Only two mathematically stronger students —Sonny and Princess— were initially able to provide an interpretation that the instructor saw as sufficiently mathematical. As an example of a mathematical interpretation, Sonny stated, "it's like where there's white people there's recess."

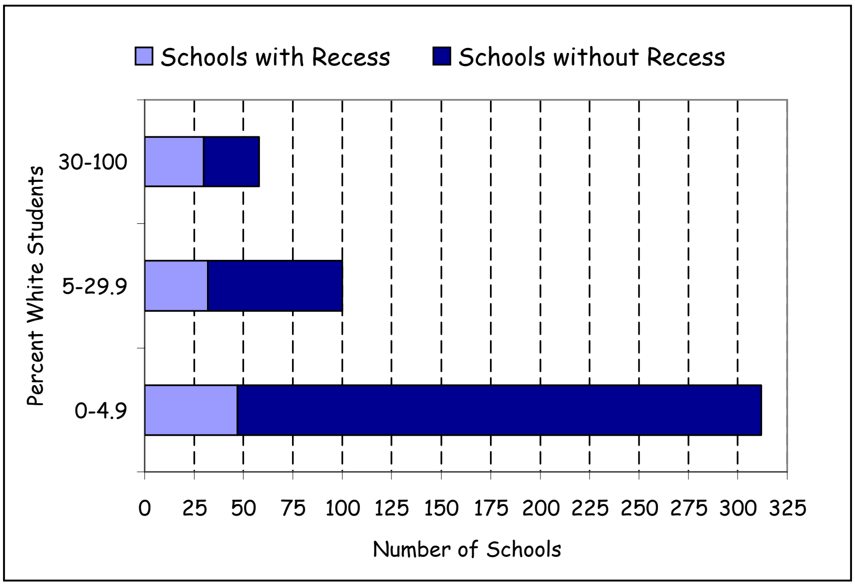

Figure 3. Race & Recess Chart from Pardo (1999)

For his part, the instructor hesitated to let the rest of the class off the hook by providing, or allowing Sonny or Princess to provide, the mathematical interpretation for them. However, after spending the first fifteen minutes of the activity vainly insisting that the majority of the class students indicate that they understood the mathematical message of the chart, he provided them with his own mathematical interpretation. He did so primarily because, at this point in the lesson, many students followed Osvaldo's lead in discussing their lived experiences in a segregated society and essentially bypassed the mathematics of the chart. In particular, after being prompted for a mathematical interpretation, Princess, an African American student, shouted, "the school is mostly black and so they don't have recess! Black kids can't go outside!" Dino, also African American, shifted the conversation away from Princess' more mathematical explanation when he responded stating, "they cause too much damn trouble, that's why they can't have recess." This launched a flurry of back and forth about the source of differential racial access to recess: problematic student behaviors, social inequality, or racist school policies.

In the ensuing conversation, a number of students gave their opinions and reacted to those put forth by their peers. Princess and Kampton, both black, pointed out that white students, such as those at Columbine High School, could "blow up shit" and still have recess, while black students were often punished for minor infractions. (It was the case that many students were bitter about being pushed into the night program from what could be seen as minor infractions.) In contrast, Dino continued to insist that black students did not behave well enough to deserve recess. He said, "they banned recess at our school cause they [i.e., African American students] was cussing up the classroom." This provoked a negative reaction from other, mostly African-American students. At the same time, if Dino's statement can be seen as fundamentally hegemonic (it blamed the victim) there was no clear articulation of a counter-hegemonic perspective (that blamed racist institutions) on the part of his classmates. Instead, many students discussed their particular experiences (i.e., whether or not they had gotten recess at their elementary school) and, at other times, shot each other's differing ideas down. To be clear, the instructor was pleased with the politicized conversation of race and recess in this CM2 activity even as he worried about the lack of a clear mathematical focus.

In sum, this CM2 lesson posed three related tensions, namely, tensions between: (1) the critical and mathematical goals of the activity, (2) the teacher valuing student contributions and stating his own "official" mathematical and political perspectives, and (3) the teacher including more data analysis activities and covering more esoteric topics required by the official geometry curriculum. There were successes as well, at least, from a CM perspective. First, the CM2 challenged urban students expectations of what content is considered appropriate for school mathematics and, more generally, school. Second, several previously disengaged students (e.g., Osvaldo, Dino, Kampton) as well as previously engaged students (e.g., Sonny, Princess) expressed interest—even excitement—about this explicitly political activity. Many night students protested when the instructor ended the politicized whole class discussion and instead asked them to write down their own opinions of the fairness of the Chicago school system. To be clear, there was also student resistance throughout this and subsequent CM2 activities. For example, at the end of this lesson, Efrain complained that CM2 was "not what we're here for" and Lucee added, "it's goofy." Other, less vocal, students quietly resisted the activity in various ways (e.g., putting their heads down, passing notes). With this example in mind, we might conclude that while CM2 provides a rupture in status quo schooling, it may not be –or is not currently– the panacea for the lack of mathematical engagement and understanding exhibited by many students of color in lower SES urban schools.

CONCLUSION

Our two examples raise questions about the nature of critical mathematics, its pedagogy, and its content; and what these means for student learning, for student, teacher, and researcher empowerment, and for social justice. Particular questions we consider are: How does the teaching context such as historical, national, and institutional influence critical possibilities and, ultimately, what counts as CM? How are students, and particularly marginalized students, positioned in CM lessons and how do they respond to that? Why are we not doing CM —CM2 in particular— with privileged students? What messages (e.g., about the nature of mathematics, mathematics learning, marginalized students, and their communities) are being sent to students in CM1 and CM2 activities? What language or discursive registers (students' vernacular, academic mathematics) or modalities (iconic, indexical, symbolic) are privileged in different versions of CM and how do they affect students' access proficiency in academic mathematics and their participation in movements for social justice?

Politically, questioning and interrogating dominant pedagogical practices remains an important task of critical mathematics educators. Critical mathematics educators may argue for the primacy of so-called real-world, politically oriented applications, what we call CM2 in this paper, for involving underrepresented groups in mathematics and for advancing social justice (Gutstein, 2005). Though this position merits serious attention, nonetheless, it is important that critical mathematics educators create narratives that are alternative to the idea that mathematics is inaccessible and the misconceptions about its inherent nature and about who can do mathematics. CM educators should not be satisfied with engaging historically marginalized students in politicized investigations of injustices (e.g., wage distributions) if they do not have access to academic mathematics. Students who have inherited the academic space opened by Melba Patillo Beals and her fellow students should not be educated under a practice that may unwittingly support an ideology that posits the oppressed as incapable of being motivated by the abstract nature mathematics.

NOTES

1. http://teacher.scholastic.com/barrier/hwyf/mpbstory/melchat.htm, emphasis added.

2. **http://www.timesonline.co.uk/tol/news/uk/article2677098.ece**

3. For a discussion of what we mean by "challenging mathematics" and for other examples, see Powell et al. (in press).

4. Although Rohan writes the interval as $0 \leq \; x \; \geq 15$, in an interview, he demonstrated that he knows well how to use inequality notion to indicate intervals on the real line.

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