RELEVANCE AND ACCESS IN UNDERGRADUATE MATHEMATICS: USING DISCOURSE ANALYSIS TO STUDY MATHEMATICS TEXTS

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Despite considerable efforts to promote access to Mathematics and to improve the mathematical performance of all students in South Africa, the reality is that fourteen years after our first democratic elections the experiences of students remain inequitable. In this paper I report on the use of discourse analysis to study the text of a mathematics problem which is used in a first-year university Mathematics course designed to give students access to tertiary study in Science. I use the method and tools of Gee (2005) to identify and explain (a) how the text presents the activity of answering a mathematics problem, and (b) how the text may position the student. I argue that this analysis raises questions about the concepts of relevance and access in undergraduate mathematics.

INTRODUCTION

Mathematics education reform in South Africa

Reform in mathematics education in South Africa since 1994 has been influenced both by reform initiatives elsewhere in the world and by the particular needs of the developing country. For example, the social justice agenda has focused on the urgent need to redress the inequities of the past, while an economic agenda has promoted the need to accelerate economic development. Reform has been characterised by calls to promote access to the subject of Mathematics and to increase the relevance of the subject for students. One way in which the term "relevance" has been contextualised in mathematics classrooms is through the use of mathematics problems with real-world contexts, which I refer to as "real-world problems" in this paper. Tertiary institutions have responded to the challenge of providing "access" by establishing foundation programmes and extended curricula, designed to provide access for students who have been educationally disadvantaged in the school system and who are identified as having the potential to pursue further studies.

Furthermore, mathematics education reform in South Africa has taken place within the setting of often rapid changes in the education system as a whole, and in wider society in general. This is a complex setting; in the past educational experience, language, class, race and poverty were often conflated, yet ongoing change, for example in the schooling system, means that these relationships are not as clear as in the past (see for example Bangeni & Kapp, 2007).

Despite the varied attempts to improve mathematics education in South Africa since 1994, empirical data shows that the experiences of students currently completing school remain different and inequitable (South African Human Rights Commission,

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2006). Furthermore, while students may gain access to study at tertiary institutions, the success rates at these institutions, particularly in Science and Engineering, remain poor (Scott, Yeld & Hendry, 2005).

From a theoretical perspective there is a wealth of work, particularly on school mathematics, that problematises attempts to make mathematics relevant (see for example Ensor, 1997). Of particular interest in this paper is recent work arguing that the use of real-world problems may prevent access to both the real-world and to the study of mathematics (see for example Dowling, 1996). There are also suggestions that certain students may be marginalised from school word problems (Cooper & Dunne, 2000; Tobias, 2006). Furthermore, certain theorists have problematised the notion of access itself. From the perspective of learning mathematics, Baker (2005) argues that formal education actually conflicts with an access agenda; while pedagogic practices may be aimed at increasing access, there is a wealth of literature suggesting that they in fact privilege certain students and reproduce social difference. This is confirmed by the research of Lerman and Zevenbergen (2004) and Cotton and Hardy (2004).

This paper

In providing the setting for this paper I have described the dynamic and challenging landscape in which mathematics education is both practised and researched in South Africa. The work presented in this paper is one attempt to investigate and explain some of the challenges. This work is located in a first-year university Mathematics course at a South African university (which I will refer to as the "Course" from this point). This Course forms part of an extended curriculum programme which is specifically designed to provide students disadvantaged by the schooling system with access to tertiary studies in Science. The majority of the students taking this Course are Black and Coloured students. The Course is taught in English, which is an additional language for more than half of the students (Visser, 2006). The Course material contains a number of real-world problems, where by "real-world" I mean "everything that has to do with nature, society or culture, including everyday life as well as school and university subjects or scientific or scholarly disciplines different from Mathematics" (The International Commission for Mathematics Instruction, 2002, p.230). I am involved in this Course in various ways; as lecturer, course convener, student advisor, and more recently as researcher.

In this paper I present the analysis of three related texts which form part of the Course material used in this access Course. I use Gee's (2005) method for discourse analysis as well as his concepts of the seven building tasks, intertextuality, situated meaning, Discourse and social language to link the features of the texts to wider social practices. These concepts are used to identify the enacted activities and identities in the texts, and to examine how the texts may position the student. This work forms part of a larger study in which I am investigating the nature of a selection of real-world problems from the Course, and examining how the practices used by students to solve these problems may be enabling or constraining.

CONCEPTUAL FRAMEWORK

Language and Mathematics as Social Practices

In this study I have adopted a perspective that language-use is a social practice (Gee, 2005). From this perspective, language is not value-free and simply a grammar and set of rules for how to use this grammar. Rather, language is linked to the context in which it is used, and language forms take on meaning in particular contexts. Consistent with this perspective on language is the view of Mathematics as social practice. Mathematics is not viewed as skills-based and divorced from contexts, but is learned and used in social contexts (Baker, 1996).

Discourse and text

I use a broad conception of the notion of discourse, as suggested by Gee (2005) who uses the term “Discourse” to include both language and non-language forms of discourse. He argues that in a social setting we use language, behaviours, actions, tools, etc. to recognise ourselves and others as belonging to a particular group or set of practices, or Discourse. At the same time we give meaning to that Discourse by reproducing or transforming it. Discourses are not mutually exclusive and fixed, but can overlap, can be contested and can change over time.

From this perspective a written text may be part of one or more Discourses, and hence may be overlapping, dynamic, and contested. Furthermore, a text has a history. In the context of education, Apple (1996) argues that, as an artefact of curriculum, a text cannot be neutral. Herbel-Eisenmann and Wagner (2007) argue that in constructing a text, a writer makes conscious and unconscious choices, and that textbooks as examples of text have agency with respect to how they can structure relationships. It should be noted here that my interest in this paper is in the text as a social artefact. I regard the detailed analysis of the text as one possible way to study how users are positioned by the text, and I am interested in the possibility for a text to reproduce or indeed produce social practice. In this paper I do not deal with how the text is used in practice, but this forms part of my wider study.

METHODOLOGY

Given the assumption that text as an aspect of Discourse is a social practice, how does one relate features of written text to the wider social practices? Discourse analytic frameworks have been used to study how texts construct roles for and position users of mathematics texts (Herbel-Eisenmann & Wagner, 2007; Herbel-Eisenmann, 2007) Bennie (in press) has used Fairclough’s three-dimensional framework as a model for linking the written text of a mathematics problem to wider discursive and social practices.

In this paper I present the analysis of text using Gee’s (2005) method for discourse analysis and a selection of his analytic tools. The initial analysis of the features of the text is done in two ways, as suggested by Gee (2005, p.54-58). “Form-function” analysis allows me to focus on the meanings communicated by particular textual features, for example, the layout, repetition, naming, etc. “Language-context” analysis enables me to study the specific meanings that different language forms take on in a particular context. Gee (2005, p.59) notes that in a specific context, a language form will take on a particular meaning, called a “situated meaning”. Certain features associated with a language form will be grouped together in a pattern, a pattern that a specific group of people find significant.

Secondly, Gee’s (2005) concept of the “seven building tasks” provides me with a systematic way to investigate how the textual features give meaning to the text. He argues that when we use language we build a “reality” by building seven things, which I give in italics in the following description. Gee claims that a situation in which language is used will involve *activities* in which people take on certain *identities*, develop *relationships* with one another and use certain *sign systems and forms of knowledge*. In such a situation certain things are given *status* and people and things take on meaning and *significance* and are *connected* or not connected to one another. I make use of Gee’s (2005) list of 26 questions about the seven building tasks; he argues that the extent of the convergence of the answers to these 26 questions can be used as one measure of the validity of the analysis.

Thirdly, how does one explain this “reality” in the context of wider social practice? Gee (2005) claims that different people will have differential access to identities and activities, and different value and status will be assigned to these identities and activities. Gee provides a number of tools that can be used to study how this may be done through language, two of which I have already described, namely “Discourse” and “situated meaning”. In this analysis I also use the concept of “intertextuality”; Gee (2005) argues that when we use language our words may reference other texts, either directly by quoting or indirectly by alluding to them. Lastly, the term “social language” is used by Gee (2005) to refer to the language aspects of a Discourse.

THE TEXTS

The focus of the analysis is on the mathematical problem in Figure 2 (which I will refer to as the “car problem” from this point). Students on the Course are required to solve this problem during an afternoon workshop session in which they work in self-selected groups of four to five students. When analysing the car problem it was decided to include two other related texts in the analysis; the textbox located immediately prior to the car problem in the Course material (see Figure 1), and the worked solutions for the car problem (see Figure 3). These worked solutions are provided to students a few days after tackling the problems in the workshop session. The three texts analysed in this paper (Figures 1 to 3) are given below in the order in which students encounter them.

The following questions are related rates problems. These MUST be set up correctly. Follow these steps for **EVERY** question:

1. Draw a diagram and define variables.
2. Write down what is given, using the correct notation.
3. Write down what is to be found.
4. Write down a formula linking the variables.
5. Differentiate and complete the question.

Figure 1. The textbox (Workshop 15, 2007 Resource Book)

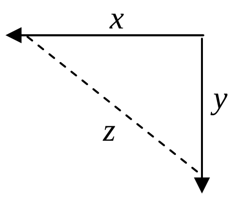
2. Two cars start moving from the same point. One travels south at 100km/h and the other travels west at 75km/h. At what rate is the distance between the cars increasing two hours later? (Let the distance between the cars after a time t be z km).

Figure 2. The car problem (Workshop 15, 2007 Resource Book)

Let x = distance covered by car A

Let y = distance covered by car B

Let z = distance between car A and car B



Given: $\frac{dx}{dt} = 75$ and $\frac{dy}{dt} = 100$

To Find: $\frac{dz}{dt}$ when t = 2 hours

$x^2 + y^2 = z^2$ (Pyth)

$\therefore 2x.\frac{dx}{dt} + 2y.\frac{dy}{dt} = 2z.\frac{dz}{dt}$

When t = 2 hours, x = 150 km and y = 200 km and z = $\sqrt{150^2 + 200^2}$ = 250 km

$\therefore 150 \times 75 + 200 \times 100 = 250.\frac{dz}{dt}$

So $\frac{dz}{dt} = \frac{1}{250}(150 \times 75 + 200 \times 100) = 125$ km/h

Figure 3. The worked solutions for the car problem (Solutions to Workshop 15, 2007)

FINDINGS

In this section I identify and describe the activity enacted in the three texts. I then explore the enacted identities in the texts and how the activity may position the student. I link the texts, via the concept of intertextuality to other texts, and via situated meaning, social language and Discourse to wider social practices. I present the textual evidence for my claims (although space prevents me from providing all the detail).

In relating the three texts to wider social practices I have named certain Discourses, for example, the “Access Discourse” and the “Everyday Discourse”. This process of naming and classifying has the effect of fixing these Discourses in time and space and of setting up boundaries. Yet, by definition, Discourses are overlapping, changing, and may vary across communities. This dilemma is noted by Moschkovich (2007) in her discussion of the naming of what she calls “Discourse practices”. Furthermore, my classification is influenced by my understanding of the setting of the study, and unavoidably reflects some value judgements about this setting. Acknowledging these difficulties, I have thus attempted to make explicit the criteria I have used in naming the Discourses used in the analysis.

I use the term “First-year Undergraduate Mathematics Discourse” to describe the mathematics currently studied by first year undergraduate students in mainstream mathematics courses, for example, at the institution at which this study is being conducted. This Discourse can be linked to a wider “Undergraduate Mathematics Discourse”, which describes the practices we would like students to participate in during their undergraduate study in Mathematics. I use the term “Calculus Reform Discourse” to describe a particular approach to teaching undergraduate calculus. This approach emphasises understanding of concepts, flexibility in moving between different representations, and the solving of problems with everyday and disciplinary contexts. The “Access Discourse” is the term I use to describe the actions, beliefs, etc. associated with attempts to provide students with access to tertiary study. My naming of the “School Mathematical Word Problem Discourse” is based on the work of Gerofsky (1996) who classifies school word problems as a particular genre. My use of the term “Everyday Discourse” is based on the work of Moschkovich (2007, p.27) who gives the name “everyday” to the practices that adults and children engage in out of school (or out of university for this study) and out of professional mathematics.

The enacted activity

I argue that the main activity enacted in the three texts is the solving of “related rates problems”. This activity is announced in the form of a statement in the first sentence of the textbox in Figure 1, and is given value by its framing in the textbox. The text of the car problem can thus be linked via intertextuality to a particular set of problems in the First-year Undergraduate Mathematics Discourse. The term “related rates problems” takes on a particular situated meaning in this Discourse and refers to group of problems that share certain characteristics, for example, they are structured in a certain way, they deal with the concept of rate of change, and they can be solved using certain procedures.

The sub-activity of solving a related rates problem can be identified in sentence 2 of the textbox; it involves "setting up" the problem "correctly" by following five steps. This sub-activity is given value as it is communicated as an instruction to the student in sentences 2 and 3, and through the use of upper case and bold letters for emphasis, for example "MUST" and "**EVERY**". These instructions can be linked via intertextuality to the text of undergraduate calculus textbooks (texts that form part of the First-year Undergraduate Mathematics Discourse), where these steps for solving related rates problems are commonly presented. The word "correctly" in sentence 2 of the textbox takes on a situated meaning in this Discourse and is given meaning by the five steps provided in the textbox.

The five steps in the textbox are in the form of instructions on different actions to be performed in solving the car problem. Certain words and phrases in these steps take on situated meanings, and are given meaning implicitly by their link to "related rates problems" and more explicitly by their link to the text of the worked solutions to the car problem (Figure 3). For example, the "diagram" required in step 1 is a **mathematical** diagram, and the "correct notation" in step 2 means using the appropriate mathematical notation for rates of change. Furthermore, certain phrases and words used in the textbox, for example, "variable" (steps 1 and 4), "formula" (step 4) and "differentiate" (step 5) are words that take on certain meanings in an Everyday Discourse, but also form part of the social language of the First-year Undergraduate Mathematics Discourse.

The car problem may appear at first to be unstructured, in the sense that the student is required to answer one question (posed in sentence 3), the solving of which requires the student to perform a number of unspecified interim steps. Yet the presence of the textbox prior to the car problem provides the student with structure for solving the problem. Instructions on problem-solving steps for solving related rates problems such as those given in the textbox are a common feature of undergraduate calculus textbooks. Yet there are certain features of these particular texts that set them apart from those traditionally found in these textbooks. Firstly, sentence 4 of the car problem contains a hint to the student about how to assign variables. Secondly, the textbox contains repeated reminders to the student about how to proceed, for example, with the use of the words "MUST", "**EVERY**", and "Write down". I argue that these features link the texts to the Access Discourse, as students are provided with additional reminders and support.

What does the text indicate about the nature of the activity identified as solving "related rates problems"? Firstly, the classification of the group of problems as "related-rates problems" suggests that certain problems in this Discourse can be grouped together according to their characteristics. Secondly, the insistence that the student follow the given five steps when solving the “related rates problems” suggests (a) that a systematic problem-solving approach is valued, and (b) that all the problems can be and should be solved in the same way. Thirdly, the five steps in the textbox, together with the worked solutions for the car problem, make it possible to identify that certain types of activity are valued, for example, presenting a full, neat, and systematic written solution, converting flexibly between different mathematical representations, carrying out certain mathematical procedures like differentiating, and having some conceptual understanding of the notion of instantaneous rate of change. The emphasis on both conceptual understanding and procedural proficiency can be linked to what is valued in the Calculus Reform Discourse.

The context for the car problem is the motion of cars, suggesting a link to the Everyday Discourse. However, as argued so far, the focus of the question is on solving a “related rates problem” within the First-year Undergraduate Mathematics Discourse. So the car problem should be read in a particular way, as only certain aspects of the Everyday Discourse are valued. A number of features of the text reinforce this argument. Firstly, there is a resistance to naming the cars and the starting point of the motion, suggesting that some of the detail about the real-world context is not important. Secondly, it is highly unlikely in an everyday setting that two cars would drive at a constant speed and at right-angles to one another. These features of the text of the car problem link this problem to a group of texts that belong to the School Mathematical Word Problem Discourse. In her discussion of the school mathematical word problem genre (which in Gee’s terms would be the social language of a wider Discourse), Gerofsky (1996, p.40) argues that these word problems only “pretend *that* a particular story situation exists” and that the story in such a problem has no truth value.

This location of the car problem text in the School Mathematical Word Problem Discourse is further reinforced by a study of the use of tenses in the text. There is a lack of consistency in the tenses of the car problem, which Gerofsky (1996) argues is a common feature of the genre. Furthermore, the composition of the problem is also typical of this genre; the first two sentences provide the “set-up” for the story and the “information” needed for solving the problem, and this is followed by a question in sentence 3 (Gerofsky, 1996, p.37). Gerofsky claims that in some cases the “set-up” is not essential to the problem solving. I argue that in the car problem, the cars could quite easily have been replaced by runners, cyclists, or walkers, with the problem remaining a “related rates problem”.

The enacted identities

I begin by discussing how the “successful student” may be positioned, that is, the student who solves the car problem as required by the textbox and the worked solutions. Firstly, the successful student is positioned as a student who has access to the First-year Undergraduate Mathematics Discourse. Such a student will have access to the situated meanings in the text, for example, knowing the pattern associated with "related rates problems" and hence being able to recognise these problems in the Course material. S/he will also have access to the situated meaning of terms such as "correctly", "diagram", "correct notation". The use of terms with particular meanings in the social language of the First-year Undergraduate Mathematics Discourse, for example, "variables" and "differentiate" positions the student as someone who understands and can use this social language.

Secondly, in order to arrive at a correct answer the successful student is required to demonstrate both conceptual understanding and proficiency in mathematical procedures such as differentiation, which can be related to the Calculus Reform Discourse. Thirdly, the successful student is required to deal with the real-world context of the cars appropriately, that is, s/he must choose only those aspects of the Everyday Discourse that are appropriate for a problem that is located in the First-year Undergraduate Mathematics Discourse. The successful student thus needs access to the assumptions of the School Mathematical Word Problem Discourse.

Continuing on the theme of the successful student, one could possibly argue that the presence of the problem-solving steps in the textbox construct this student as a problem-solver in mathematics, and as a student who can solve a real-world problem by mathematising it and using appropriate mathematical tools. However, I argue that certain features of the text may construct an identity for the student that conflicts with the identity of the successful student described so far. Firstly, the student is instructed to solve the "related rates problem" in a certain way, when the method presented is not the only possible method for solving the car problem. Secondly, the student is repeatedly reminded (with textual features such as upper case letters, bold and underlining) to follow these steps. Thirdly the student is presented with a hint for starting out with the problem (sentence 4 of the car problem). I argue that these textual features construct the student (a) as someone who needs help solving the car problem, and (b) as a student who does not usually do what is required when instructed to follow given problem-solving steps. The student is thus positioned as an "access student".

CONCLUSION

This analysis is restricted to the car problem and the two associated texts. Yet a detailed discourse analysis of these three texts raises a number of questions related to the possible positioning of the student and the concepts of access and relevance. These questions need to be considered in the wider study in which I am investigating the practices used by students when solving the car problem as well as other real-world problems.

Regarding the concept of access, I begin by asking two questions: "Does classifying problems in a group as 'related-rates problems' and suggesting that they can all be solved in the same way promote access to the Undergraduate Mathematics Discourse?" and "Does providing the student with problem-solving steps and a hint for getting started on the problem promote access to the Undergraduate Mathematics Discourse?” Secondly I ask, “Who has access to the social language of the First-year Undergraduate Mathematics Discourse?” Research evidence suggests that students learning in English as an additional language may be at a disadvantage as they face the dual challenge of mastering the language of instruction as well as the social language of Mathematics (Setati, 2005; Barton et al. 2005).

Thirdly, I ask, “Who has access to the assumptions of the School Mathematical Word Problem Discourse?” I have argued that while the problem may draw on aspects of the Everyday Discourse, only certain aspects of this Discourse are regarded as being important, and these are the aspects required to solve a “related rates problem” located in the First-year Undergraduate Mathematics Discourse. The term “relevance” thus takes on a particular meaning, that is, as relevance in this particular Discourse. I ask, therefore, “Who has access to what is relevant in this Discourse?” This question needs to be asked, given claims by Cooper and Dunne (2000) that successful performance on school word problems can be linked to social class. Given the varied nature of schooling in South Africa, as described in the introduction to this paper, it is possible that those students entering tertiary institutions will have had very different experiences of school mathematical word problems.

Lastly, I ask, “Do the successive reminders to the student and the positioning of the student as an access student promote access to the Undergraduate Mathematics Discourse?” Assuming that students have agency with respect to the identities that they choose to inhabit, it is possible that some students may make a conscious choice not to follow the repeated instructions in the car problem.

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