DANCING A VIENNESE WALTZ TO WITTGENSTEIN’S NOTES

A REACTION TO OLE RAVN CHRISTENSEN’S ADDRESS

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PRELUDE

The concise, however quite informative, presentation of later Wittgenstein’s philosophy of mathematics offered by Ole Ravn Christensen in his address, accompanied by his personal claims about its potential impact on mathematics education research, may actually open the “Aeolus’ bag” in the “Odyssey” of our discussions[1].

According to the social reading of Wittgenstein proposed by Ole, this philosophical perspective on mathematics “emphasises a number of cluster concepts, including: anti-foundationalism; ·anti-essentialism; anti- or post-epistemological standpoint; anti-realism about meaning and reference; suspicion of transcendental arguments and viewpoints; rejection of the picture of knowledge as accurate representation; rejection of truth as correspondence to reality; rejection of canonical descriptions and final vocabularies; suspicion of meta-narratives” (Peters, 2002). On the other hand, Wittgenstein’s philosophy of mathematics, inextricably woven with his philosophy of language, is founded, almost exclusively, on rejections and it may not be positively associated with any philosophical doctrine or position.

It is obvious that numerous issues arise from this starting point concerning mathematics education, considered as a whole or in its many aspects of research and practice. Therefore, although many issues may be brought forward in our discussion, I will briefly put in the following three, plus a forth, questioning notes, analogous to a “three-fourth time Viennese waltz” on Wittgenstein’s lyrics “orchestrated” for this meeting by Ole’s address**.** Questions which are possibly not clearly or even not completely being formulated.

NOTE ONE: WHAT ABOUT THE STATUS OF MATHEMATICS?

The adoption of a social constructivist standpoint about mathematical knowledge and its objects, in contrast to the ontological assumptions of Platonism and mathematical realism, which is a standpoint more or less clearly adopted by Wittgenstein, on top of raising various particular issues, it makes, in principle, the fundamental status of mathematics as a scientific discipline ostensibly problematic (Chassapis,1999). Because mathematics is, as a scientific discipline, a sector in the quest for understanding our experienced reality; and consequently, if that conception is given up or radically modified, then mathematics seems, in my view, to run the risk of being reduced to a kind of intellectual play, carried out mostly in the realm of discourse. If we accept that an ontology of mathematics is nonsense or exclusively

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produced or constructed by the discursive realm of mathematics, or that the ontology of mathematics is created by its use, then mathematical discourse, beyond its being self-referential, constitutes a self-enclosed arbitrary system and mathematics as a discipline does not provide a theoretical knowledge of any kind of reality existing independently and outside of itself. As Wittgenstein claims, mathematics by itself does not deal directly and precisely with any realm of empirical reality, which may provide the necessary grounds for confirming or refuting its propositions.

Therefore, where the issue of mathematics as a discipline is concerned, it seems that its conceptual knowledge and the objects it appropriates coincide and any reference mathematics makes to an independently and outside of itself existing reality, appears possible only by way of its mediation by the other scientific disciplines which utilize mathematics. If so, and from a disciplinary viewpoint, the constitution of the particular scientific methods (calculi and proofs) which mathematics employs is also rendered problematic in many ways, since it is supposed that these methods, beyond any other application, correlate mathematical knowledge to its objects and simultaneously test the efficacy of that knowledge. Given the outlined impact of Wittgenstein's conception on the status of mathematics as a scientific discipline, what may be the content of school mathematics, beyond its organising modes dictated by versions of Platonism?

NOTE TWO: WHAT ABOUT THE STATUS OF PROOF IN MATHEMATICS?

Calculus and proof may be proclaimed the main production methods of mathematical knowledge, and even considered to be the socially established rules of the mathematics language-games. Any mathematical "proof", in particular, transforms, through its production, a hypothesis into a mathematical proposition, expressing or describing a mathematical "fact" or "event", however defined. That is, every "proof" produces mathematics. At the same time as producing mathematical "facts" or "events", a "proof" produces itself, and, by its resultant existence, validates retrospectively the capacity of a mathematical system to produce those particular "facts" or "events". In other words, the existence of a "proof" validates in itself the capacity of the relevant mathematical system to appropriate conceptually the specific "fact" or "event" conjectured by the hypothesis, while it is the "proof" itself, which constitutes that validation.

In this sense, the production of mathematics and the validation of the capacity of mathematics to produce mathematics are coincident procedures. This is not the case in other disciplines, such as, for instance physics or chemistry, where the procedures of any proof are exclusively subjected to a conceptual system, in contrast to the procedures of validation, which are completely subjected to the scientific methods, experimental as a rule, employed to associate the conceptual system with its objects of study. For these, among other, reasons, the status and function of "proof" either established on a deductive or on a quasi-experimental logic are unique in mathematics. What may be the corresponding status and function of “proof” in school mathematics from a Wittgensteinian viewpoint? A language-game producing meaningful words and sentences and at the same time validating the capacity of the rules of that game to produce these words and sentences as well as their meanings according to its use?

NOTE THREE: WHAT ABOUT THE REFERENCE OF MATHEMATICS TO REAL WORLD SITUATIONS?

Mathematics provides us with “different grids or structures by which we measure or describe the world”, “measures in the sense that they set up the rules of grammar through which we can describe” and “some rules of grammar are presupposed in any description of reality”. Using these words from Christensen’s reading of Wittgenstein we may recapitulate his conception of the references of mathematics to real world situations. Bearing in mind, of course, that mathematical objects acquire their meaning from the rules attached to them, according to which they are used; otherwise they do not stand for anything material, abstract or symbolic.

However, is not such a conception problematic, at least for school mathematics? In the sense that it confuses mathematics with its uses and furthermore it potentially eliminates a necessary, in my view, distinction, between mathematical structures and mathematical functions, reducing mathematics to mathematisations, which is one of its constitutive aspects. The other aspect being the various mathematical systems, which play the role of “structures by which we measure or describe the world” producing by these uses of mathematical systems various mathematisations of real world situations.

Is not a productive combination of mathematical structures with their references to real world contexts an aim of mathematics education and a field of relevant ventures? How can we manage the pursuit of such an aim, if we start from a Wittgensteinian perspective?

NOTE FOUR: MATHEMATICAL ACTIVITY IS A SOCIAL ACTIVITY, BUT SOCIAL IS NOT …

Hersh in his book “What is mathematics, really?” prompts us: “*From the viewpoint of philosophy mathematics must be understood as a human activity, a social phenomenon, part of human culture, historically evolved, and intelligible only in a social context.*” (Hersh 1997, p. 11). In other words, to understand mathematics as a human activity means to frame it in a social context, to embed it in history, to consider its cultural attainments and, I would add, to take into account its particular political functions. However, mathematics as well as mathematics education, as human activities involve actors who act, aiming at the accomplishment of particular tasks, who use tools and artifacts, who work in communities under rules, conventions and statutes, who develop particular forms of collaborations, who are subjected to specific divisions of labour. All these factors are permeated by their contradictions forged by their particular conditions of development and characterised by their own peculiarities.

Against such a complex dynamic system as a social activity may be, mathematics appears in the Wittgensteinian approach as a network of different language-games consisting in procedures for the manipulation of language according to established rules, therefore as a primarily socio-linguistic activity, subsuming to the discursive every aspect of the social. Letting aside many questions concerning the power and control of language, I wonder how this approach can cope with the multiplicity of factors shaping a social activity such as mathematics, or mathematics teaching and learning, as well as their research related fields.

FINALE

Many years ago, Steiner claimed that a philosophy of mathematics has powerful implications for mathematics education, pointing out that: “Concepts for teaching and learning mathematics – more specifically: goals and objectives (taxonomies), syllabi, textbooks, curricula, teaching methodologies, didactical principles, learning theories, mathematics education research design (models, paradigms, theories, etc.), but likewise teachers’ conceptions of mathematics and mathematics teaching as well as students’ perception of mathematics – carry with them or even rest upon (often in an implicit way) particular philosophical and epistemological views of mathematics” (Steiner, 1987, p. 8).

The argument of a close association of a philosophy of mathematics with fundamental features of mathematics education, although widely accepted, has been principally developed on the grounds of theoretical analyses. Taking this association for granted, Christensen proposes a particular reading of later Wittgenstein’s philosophy of mathematics, as favouring a social direction for our research and practice on mathematics education, based also on the ground of a theoretical analysis. In practice, however, is it feasible to make this social perspective productive without undermining the disciplinarity of mathematics and without liquidating the teaching and learning of mathematics in schools? We may ask “what for?” but let this question out of the discussion, at least for the present.

NOTES

1. In Homer's epic *The Odyssey*, Aeolus gives Odysseus a leather bag of winds with instructions to keep them bottled up for a safe trip. But his crew lets them out of the bag, sending them into more adventures.

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